ABSTRACT

DESIGN OF PRESSURE VESSELS

Steve Papadakis

The processes necessary for the design of pressure vessels have been studied. Ferrous and non-ferrous materials are discussed as well as their behavior at sub-zero and high temperatures including their corrosion resistance.

The application of the membrane theory of elasticity is demonstrated in calculating stresses induced by internal pressure. Also; local discontinuity stresses on vessels are outlined.

Design equations which give the required wall thickness of vessels under internal or external pressure, are derived and discussed. Furthermore, design methods for typical vessel components are described. An example illustrating the design procedures in connection with Section VIII, Div. 1 of the ASME Code is provided at the end of the report.

ACKNOWLEDGEMENTS

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NOMENCLATURE

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x,y,z	space coordinates
P	design pressure, psi (kPa)
t	vessel wall thickness, in.(mm)
σ_1	meridional stress, psi (MPa)
σ_2	hoop stress, psi (MPa)
S	allowable or design stress, psi (MPa)
r,R	vessel radii, in.(mm)
α,θ	indicated angles,(°)
ρ	radius of curvature, in.(mm)
a,b	major and minor axes of an ellipse respectively, in.(mm)
M_x	bending moment, lb_f-in.(N-m)
N_x	membrane force, lb_f(N)
N_θ	membrane force at an angle θ w.r.t. a reference point, lb_f(N)
M	external overturning moment, lb_f-in.(N-m)
E	modulus of elasticity, psi(MPa)
E	efficiency of weld joints
β	attachment parameter
c	loading parameter
γ	shell parameter
D	diameter of a shell or head, in.(mm)
ν	Poisson's ratio
I	moment of inertia of a specified cross-section, in^4 (mm^4)
L_c	critical length, in.(mm)
L_s	design length between stiffeners, in.(mm)

L	crown radius for heads, in. (mm)
ε	strain
t'	equivalent thickness resulting from the combined ring-shell cross-section moment of interia, in. (mm)
A	cross-sectional area of a section, in^2 (mm^2)
I_s	required moment of inertia of a stiffening ring cross-section, in^4 (mm^4)
h	depth of ellipsoidal head, in. (mm)
r	knuckle radius, in. (mm)
c	corrosion allowance, in. (mm)
D_p	reinforcing pad diameter, in. (mm)
h_n	projecting length of nozzle beyond the inner surface of a vessel, in. (mm)
d	nozzle diameter, in. (mm)
t_r	required thickness of vessel wall, in. (mm)
t_n	actual thickness of nozzle wall, in. (mm)
t_{rn}	required thickness of nozzle wall, in. (mm)
t_e	thickness of reinforcing element, in. (mm)
W	total design load for gasket seating lb_f (N)
H_d, H_G, H_T	forces acting on flange, lb_f (N)
h_D, h_G, h_T	moment arms on flange, in. (mm)
S_H	longitudinal bending stress in the hub of flange, psi (MPa)
S_R	radial stress in flange, psi (MPa)
S_T	tangential stress in flange, psi (MPa)
S_{fo}	design stress of flange, psi (MPa)
M_T	total moment exerted at the support of a vessel, ft-lb_f (N-mm)

W_v weight of the vessel, lb_f (N)

S_c compressive stress, psi (MPa)

CHAPTER I

INTRODUCTION

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INTRODUCTION

1.1 INDUSTRIAL PRESSURE VESSELS

The chemical, petrochemical and power generation industries convert one form of materials or energy into another by chemical or physical means. These processes require the handling and storage of large quantities of materials in containers or vessels of varied construction, depending upon the application and the operational requirements.

Pressure vessels are leakproof containers, and as the name implies, their main purpose is to contain a given medium under pressure and temperature. They may be of any shape and size ranging from beer cans, automobile tires, or gas storage tanks, to more sophisticated ones encountered in engineering applications. Pressure vessels, commonly have the form of a cylinder, sphere, ellipsoid, cone or a combination of these shapes. Correspondingly, they are identified as cylindrical, spherical, ellipsoidal or conical. However, some pressure vessels are named after the type of function that they are required to perform. For example, the distillation column shown in Fig. 1.1, is a vessel used in petroleum refining processes. The heat exchanger shown in Fig. 1.2, is a vessel widely used in many types of industries to transfer heat from one fluid to another. Also, the reactor vessel shown in Fig. 1.3, which contains substances undergoing chemical reactions, is

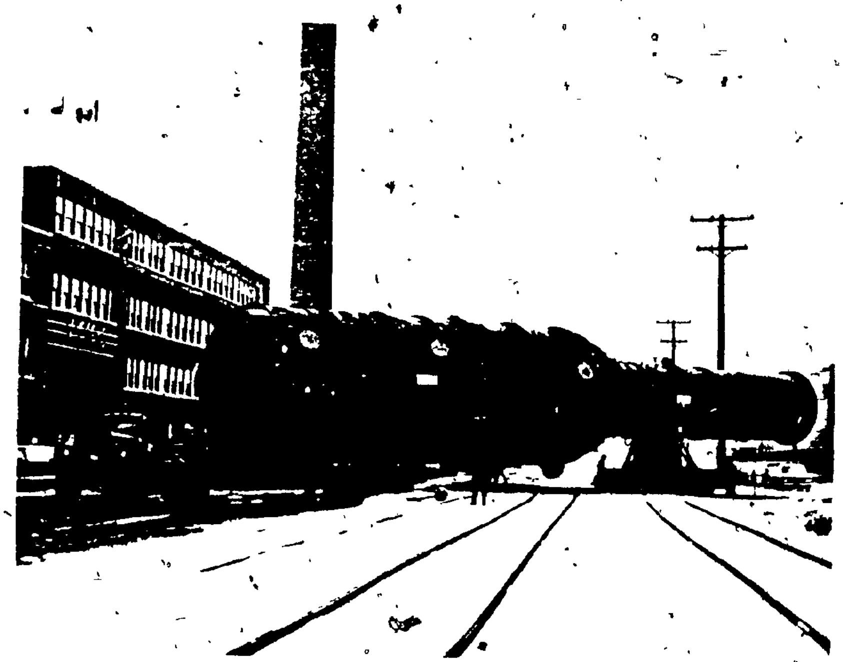

FIG. 1.1 Distillation Column.
(Courtesy of Canadian Vickers Ltd.)

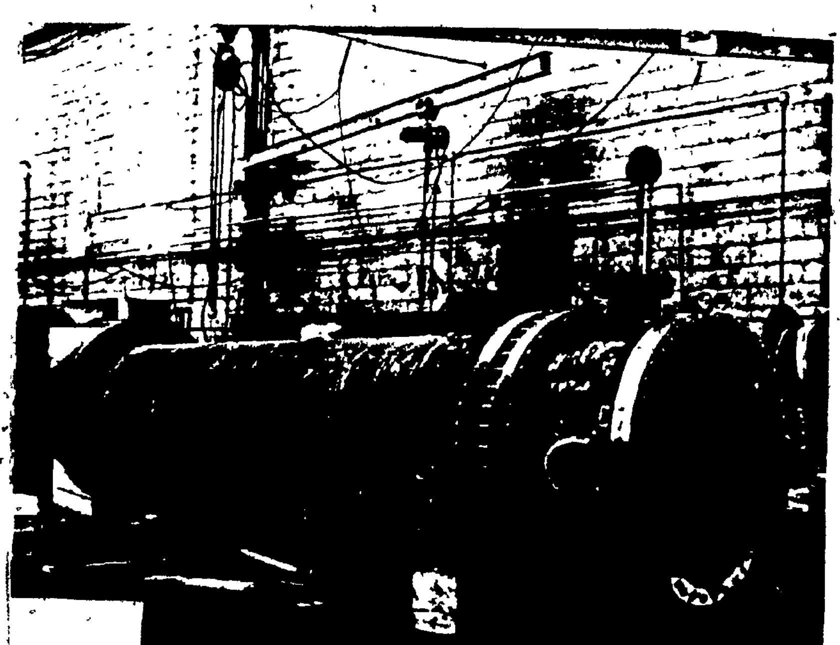

FIG. 1.2 Heat Exchanger.
(Courtesy of Canadian Vickers Ltd.)

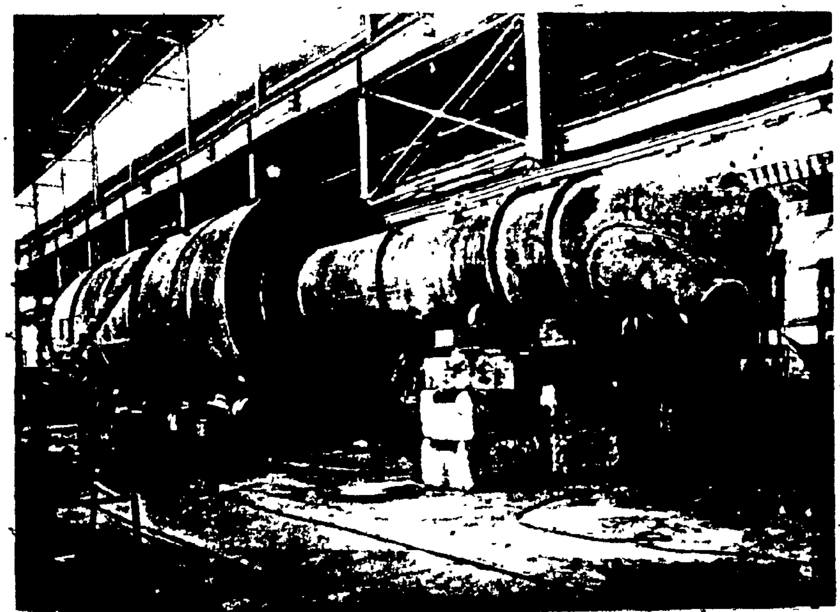

FIG. 1.3 Reactor Vessel
(Courtesy of Canadian Vickers Ltd.)

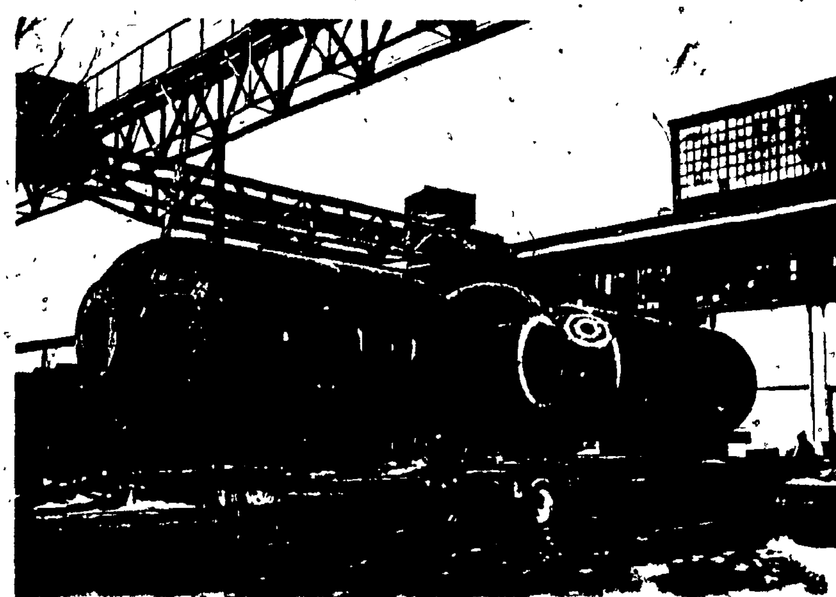

FIG. 1.4 Deep Diving Chamber
(Courtesy of Canadian Vickers Ltd.)

used in petrochemical industries and the deep diving chamber shown in Fig. 1.4 is a vessel used in ocean exploration studies.

1.2 DESIGN CODES

Because of safety implications and environmental hazards arising from the operation of pressure vessels, there is an obvious need to standardize engineering and fabrication practices. To assure minimum performance standards, several design codes have been enacted. Wherever these codes have no legal standing it is not unusual practice for the owner of the pressure vessel to impose the use of pertinent codes.

In Europe, the most widely National Codes are [1]*:

GERMANY	-	Arbeitsgemeinschaft Druckbehälter (AD - Merkblätter)
FRANCE	-	Syndicat Nat. de Chaudronnierie et Tolerie (SNCT.No.1)
BRITAIN	-	British Standards Institution (BS 1500, BS 1515)
ITALY	-	Controllo della Combustione e Apparecchi a Pressione (CCAP)

In the United States and Canada, the most widely used standard is the ASME Boiler and Pressure Vessel Code, published by the American Society of Mechanical Engineers (ASME).

*Numbers in brackets designate references at the end of the report.

Since the ASME Code is a legal document in all Canadian Provinces in the design of pressure vessels, its development will be summarized in the following sections.

The need for a code became apparent when a disastrous boiler explosion occurred on March 10th, 1905, in a shoe factory located in Brockton, Massachusetts [2]. As a result of this explosion, 58 people were killed, 117 were injured and property damage was estimated at $250,000. For years prior to 1905, boiler explosions had been regarded as either an inevitable evil or 'an act of God'. Following this accident, the Commonwealth of Massachusetts introduced the first legal code of rules in 1907, covering the construction of steam boilers. In 1908, the State of Ohio enacted similar legislation adopting, with a few changes, the rules of the Massachusetts Board.

Thereafter, other states began to formulate rules and regulations for boiler construction. As regulations differed from state to state, manufacturers and users of boilers made an appeal in 1911 to the Council of the American Society of Mechanical Engineers to rectify the situation. The Council responded by appointing a committee of seven members to develop standard rules for all types of pressure-retaining equipment. In 1913, the Committee published its preliminary report and sent 2,000 copies to engineers and experts on the field requesting comments and suggestions. One year later, after several meetings and public hearings, a final draft of the first ASME Code was issued entitled "Rules for Construction of Stationary Boilers and for Allowable Working Pressures."

Since 1914, many changes have been made and new sections added to the Code as the need arose. The sections appeared in the following order:

Section		
I	Power Boilers	1914
II	Material Specifications	1924
III	Boilers of Locomotives	1921
IV	Low-Pressure Heating Boilers	1923
V	Miniature Boilers	1922
VI	Inspection	1924
VII	Suggested Rules for Care of Power Boilers	1926
VIII	Unfired Pressure Vessels	1925
IX	Welding Qualifications	1940

To keep up with industrial growth and technological progress, major revisions and additions are continuously made to the code. New editions are issued triannually with supplements every six months. To-date, (1977 edition) the ASME Code structure is as follows:

Section	
I	Power Boilers
II	Material Specifications Part A - Ferrous Materials Part B - Non-ferrous Materials Part C - Welding Rods, Electrodes and Filler Metals
III	Division 1 and Division 2 - Nuclear Power Plant Components with Subsections NB, NC, ND, NE, NF, NG and Appendices
IV	Heating Boilers
V	Non-destructive Examination
VI	Recommended Rules for Care and Operation of Heating Boilers
VII	Recommended Rules for Care of Power Boilers
VIII	Pressure Vessels, Division 1 and 2
IX	Welding and Brazing Qualifications
X	Fiberglass-Reinforced Plastic Pressure Vessels
XI	Rules for Inservice Inspection of Nuclear Power Plant Components

Section I and Section VIII, Division 1, deal with conventional pressure vessels, while Section VIII, Division 2, provides stringent alternative rules, and Section III covers the design of nuclear equipment. Where deemed necessary in this Report, Divisions 1 and 2 of Section VIII are discussed and compared.

A detailed guide to ASME Code, Section VIII, Division 1, for the design of pressure vessels and their components is provided in Fig. A-1.

1.3 CRITERIA IN VESSEL DESIGN

Assessment of material properties and method of analytical approach are the two important criteria that govern the design of pressure vessels. In order to assure safe operation and high reliability, it is imperative that these criteria be given adequate consideration in the early stages of the design.

Material properties include tensile strength, ductility, creep and corrosion resistance. Tensile strength, also called the ultimate strength, is the maximum stress that a material can sustain without rupturing. Ductility is a measure of the deformability of a material.

Ductile materials exhibit high strain when subjected to a tensile test, as shown in Fig. 1.5, while brittle materials indicate small strain and high stress values. Brittle materials, in low temperatures and in the presence of a surface crack or notch, may fail with little or no evidence of plastic strain [3]. Brittle failure of vessel materials was a common occurrence in the past, however, with modern material technology this type of failure can be controlled.

At high temperatures, the governing criterion is the creep rate of a material. Creep is the rate of elongation of a material as a function of time under constant load. Ductility, creep and corrosion rates are discussed in Chapter II.

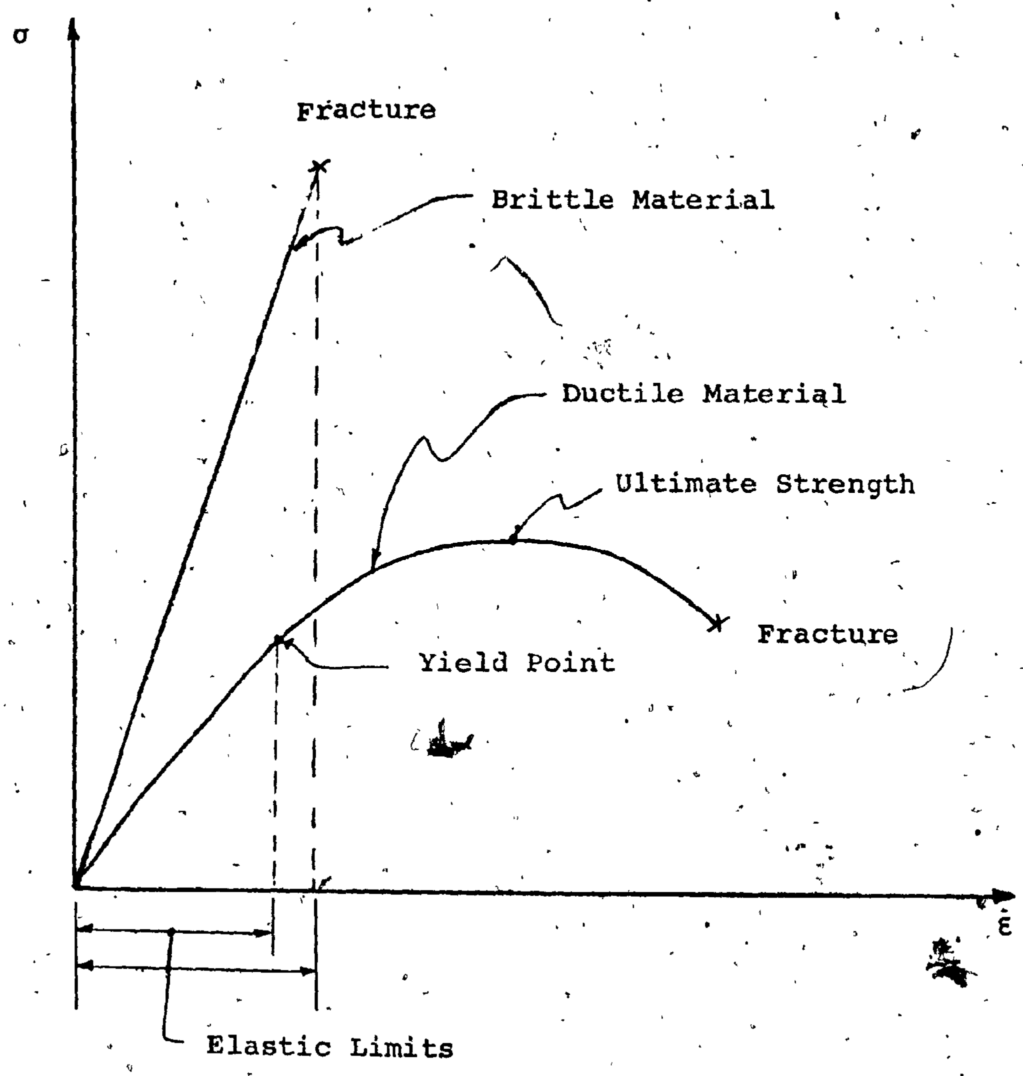

FIG. 1.5 Stress-Strain Curve of a Brittle and Ductile Material

The design approach includes the criteria on which the design stresses are derived. In Section VIII, Division 1 of the ASME Code design stresses below the creep range are based on 1/4 of the tensile strength or 5/8 of the yield, while in the creep range design stresses are based on the rupture stress or 1% creep strain in 100,000 hours. Division 2, of the same Section, covers design in the creep range and stresses are based on 1/3 of the tensile strength or 2/3 of the yield. The reason for permitting higher design stresses than in Division 1 is, that Division 2 requires a detailed stress analysis under all possible combinations of loading, whereas only direct membrane stresses are investigated in Division 1.

1.4 SCOPE OF THE REPORT

The aim of this report is to introduce the reader to the design of pressure vessels. Throughout the discussion, the design rules and criteria mentioned refer to the ASME Code.

Material properties and their influence on the design are covered in Chapter II, where both ferrous and non-ferrous materials are discussed. Chapter III demonstrates the application of the membrane theory of elasticity in calculating stresses developed by internal pressure. Also, discontinuity stresses at the head-shell junction and local areas on the vessel are outlined. Chapter IV contains a summary of common design equations which may be used to find the required material thickness under internal and external pressures. Also, design procedures for typical vessel components are described.

Finally, an illustrative example of a vessel design, based on Division I, Section VIII, of the ASME Code, is provided in Chapter V.

CHAPTER II

SELECTION OF MATERIALS

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2.1 GENERAL REMARKS

A major consideration in the design of pressure vessels, is the selection of appropriate materials to meet the requirements of each particular application. In order to arrive at the optimum selection, the designer should be fully aware of all parameters related to the intended purpose of the vessel, stipulations of relevant codes and statutory regulations, as well as cost and availability of materials.

Modern industrial applications of pressure vessels cover an extensive range of temperature and pressure. Some units are required to operate at temperatures as high as 1300°F (700°C), while others are designed to work at - 330°F (-200°C). The design pressure also varies from vacuum to as high as 20,000 psig (137.8 MPa). It is apparent that material selection will be dictated to a large extent by the service environment. For example, operation at very low temperatures requires the use of notch-tough materials, while at high temperatures creep strength is essential.

The designer should assure that the selected material does not entail any fabrication problems. This is especially true when dealing with high strength alloys where shaping and joining demand a high level of workmanship. Furthermore, a good working knowledge of design codes concerning material requirements is essential to guarantee acceptance by the concerned regulatory bodies.

In North America, pressure vessel materials are specified in accordance with the standards of the American Society for Testing and Materials (ASTM) and the American Society of Mechanical Engineers (ASME). In general, materials used in pressure vessel technology may be grouped as follows:

1) Steel, including carbon steel, low and high alloys and all types of cladding (ASME II, Part A).

2) Non-ferrous, including aluminium, copper, nickel and their alloys (ASME II, Part B).

3) Special purpose materials such as titanium, zirconium, platinum, etc., (ASME II, Part B).

4) Non-metallic such as fiberglass-reinforced plastics (ASME X).

In this Report, we will discuss the first two groups since they represent the principal materials employed in current pressure vessel construction.

2.2 STEELS

Steels, which are essentially alloys of iron and carbon, are very versatile and are used extensively in the fabrication of pressure vessels. Steel is made in two crystal forms. One form has a body-centered cubic lattice known as ferrite and the other has a face-centered cubic lattice known as austenite. The carbon content, which determines the hardness, is usually maintained below 0.35%.

Material properties depend mainly on the process by which steel is made. Steel is made by removing impurities from molten iron (pig iron) or from a mixture of iron and steel scrap. The type of impurities removed depend on whether an acid (silicon) or basic(lime) agent is employed. The most frequently used steel-making processes are: Melting, Pouring and De-oxidation [4].

In the Melting process, steel is made in Bessemer converters, electric furnaces, open hearth furnaces and basic oxygen converters. Historically the Bessemer process is the first bulk steel-making method.

In the original basic Bessemer converter, air is blown through a charge of molten iron to remove excess carbon. Manganese is subsequently added at the end of the 'blow' to counteract the brittle effect of sulfur residues. In converters with an acid refractory lining, it is not possible to remove phosphorous from the iron. Some improvements to the acid Bessemer process have been achieved, but this method of steel-making is little used in North America because of the type of impurities contained in local iron ores. Bessemer Steel may have an undesirably high nitrogen content, which results in strain age embrittlement [6].

Open hearth and electric furnaces steel, is made by melting iron and steel scrap under suitable slags. Excess carbon is removed by accelerating oxidation, through the addition of iron ore into the melt or by injecting oxygen. These processes give adequate control over the content of non-metallic impurities in the steel and are generally accepted in making boiler and pressure vessel steel plates.

In oxygen converters, phosphorus, sulphur and carbon are removed as in the Bessemer Method. In this process, as the name implies, oxygen is blown on the surface of the charge, instead of air [5]. ASTM specifications accept basic oxygen steel for boiler and pressure vessel plates.

In the Pouring method, carbon is removed from the iron ore by combining it chemically with oxygen. This results in a significant quantity of oxygen being present in the final product. When this steel is poured into a molt, the remaining carbon and oxygen react to form carbon monoxide (CO), leaving the outer part of the ingot somewhat porous and decarburized. When oxygen is reduced to a low level, the reaction ceases and the inner part of the ingot, which contains the residual carbon and most of the impurities, solidifies. Plates rolled from such ingots have pure steel in the outer layer and impure carbon steel at the core. Such steels are known as rimmed steels.

In the De-oxidation process, oxygen is removed by adding small amounts of compounds (de-oxidants) that have an affinity for oxygen. Silicon, manganese and aluminum are added in small quantities in the charge. The aluminum reacts with nitrogen to form aluminum nitrate (AlN). Those steels containing aluminum are known as killed steels. Semi-killed steels are an intermediate quality material between rimmed and killed steels.

The creep rupture strength of 'aluminium-treated' steel is inferior to that of silicon, killed or semi-killed steel at temperatures around $750^{\circ}F$ ($400^{\circ}C$). The addition of de-oxidants, such as silicon, manganese and aluminium, may create a lamellar weakness. This disadvantage can be minimized by pouring the steel under vacuum conditions. In vacuum pouring, small amounts of de-oxidants are added resulting in improved quality control.

2.2.1 Carbon Steels

The iron-carbon alloys containing up to 1.7% carbon with only minor amounts of other elements are referred to as plain carbon steels. A typical chemical composition of carbon steels used in pressure vessels is less than 0.25% carbon, about 0.7% manganese (sometimes up to 1.5% manganese), about 0.05% phosphorus and sulphur. Carbon steels with up to 0.4% carbon are used for bolts, studs and nuts [6].

Rimmed steels are seldom used in pressure vessel construction because of their lack of chemical homogeneity. Semi-killed carbon steels are the cheapest and most commonly used material for conventional light duty service vessels.

ASTM specifications for non-killed steels do not stipulate the degree of de-oxidation to be employed and the steel maker is free to supply any grade that meets the chemical and physical requirements. Fully de-oxidized silicon killed steels are more homogeneous and are used in more demanding applications.

Carbon steel may be classified as fine-grain or coarse-grain. Fine-grain carbon steel is employed in moderate and low temperature application, whereas coarse-grain steel in intermediate and elevated temperature application. This classification is accepted by both the ASTM and ASME Codes. ASTM A-516 is a fine-grain steel, which according to the ASME Code Section VIII, can be used up to $850^{o}F$ ($455^{o}C$). ASTM A-515 on the other hand, is a coarse-grain steel and can be used up to $1,000^{o}F$($540^{o}C$) [8]. Another common material is ASTM A-285,Gr.C. employed for vessels which do not contain lethal liquids or gases, and have operating temperatures between -20 and $900^{o}F$ (-30 and $480^{o}C$).

Recently, in order to meet the requirements of the nuclear power industry, a number of carbon-manganese steels with a high notch toughness have been developed. Their carbon content is below 0.15% with about 1-2% manganese [7].

Carbon steels are generally used in the normalized condition, that is, after being heat treated up to $1650^{o}F$($900^{o}C$) and allowed to cool slowly by exposure to air. Alternatively, the annealed condition may be achieved by selecting the temperature and cooling rate during rolling and forming. The material is then in the 'as rolled' condition.

2.2.2 Low Alloy Steels

Low alloy steels contain less than 10% alloy elements in total. Special properties are conferred on these steels by the presence of either one or several other alloying elements, the most common being chromium, nickel and molybdenum in varying amounts but always kept below 10%. Chromium and molybdenum improve the mechanical properties, especially at high temperatures, and enhance corrosion resistance. Nickel on the other hand increases the notch toughness at very low temperatures. A typical composition of other elements consists of 0.15% carbon, 1.0% manganese and about 0.3% silicon [6]. The phosphorous and sulphur contents are always kept below 0.05%. Low alloy steels with carbon content of 0.4% are used for bolting.

A low alloy steel, with high creep resistance around 885°F (475°C), is the 0.5% molybdenum steel which was originally developed for superheated steam. However, as a result of reported failures, it is now being replaced for high temperature service, by chromium-molybdenum steels with 0.5-5% chromium and 0.5-1% molybdenum. For low and ultra-low temperatures, 3.5% nickel steel and 9% nickel steel are used respectively [9].

It is important to emphasize that low alloy steels developed for high temperature applications exhibit a lower elongation at rupture than the ordinary carbon steels. Because of this, cracks may initiate during hydrostatic tests, which are usually conducted at room temperature, at points where high localized stresses exist. The designer must ensure that the maximum stresses in low alloy pressure vessels do not exceed the yield point during pressure testing and subsequent service time.

2.2.3 High Alloy Steels

Steels that contain more than 10% alloy elements in total, are frequently referred to as high alloy steels. There are two basic types: straight chromium ferritic steels, with chromium content ranging from 13% to 27%, and chromium-nickel austenitic steels, containing 18% to 25% chromium with 8% to 20% nickel. The carbon content for both types is maintained between 0.04% and 0.25%. [9]

The straight chromium ferritic steels are generally specified for corrosion-resistant service. 13% chromium steel alloyed with aluminum is commonly employed in petroleum refinery vessels, which are in contact with sulphur-bearing oils, at elevated temperatures. The addition of aluminum ensures that the steel is ferritic and minimizes hardening in the heat-affected zone of the welds [6].

17% and 27% chromium steels are rarely encountered in pressure vessels primarily because they are susceptible to temper-embrittlement.

Austenitic chromium-nickel steels are used mainly in sub-zero and high temperature applications as well as, in corrosive environments. Austenitic steels are nominally specified by their chromium and nickel content. For example, the designation 18/8, 10/10, 17/12, 25/12, and 25/20 refer to the chromium and nickel contents respectively, present in each alloy. A widely used high alloy steel is ASTM A-240 type 304 containing a maximum of 0.08% carbon. This material has fairly good anticorrosive properties and is suitable for both low and high temperature service. Austenitic chromium-nickel steels have excellent notch toughness and may be used for temperatures down to $-310^{\circ}F$ $(-190^{\circ}C)$ [7].

Straight chromium steels are used in the chemical industry usually for non-pressurized components. Austenitic steels are used for pressure parts as well as linings, whenever it is necessary to employ materials with high corrosion resistance and good creep strength. In high alloy steels it is essential to follow the correct manufacturing practices, especially in weld preparations and pre-or post-weld heat treatment.

2.2.4 Clad Plate

In some cases, the surface properties of steel can be improved by incorporating special surface layers. When such surface layers exceed 3% of the total mass of the base plate, the resulting product is known as composite steels. The process of adhering a surface layer to the base plate is called cladding. The materials most frequently used for cladding are

18/8 stainless steels, 13% chromium steels, nickel and nickel alloys [9].

Cladding may be accomplished in several ways some of which are: casting the clad metal around or adjacent to a previously formed steel ingot, rolling composite plates to produce bonding by pressure, welding or building up a thick surface through weld deposition. Roll bonding is limited to plates with initial thicknesses of below 2.5 in. (63.5 mm). Thicker clad sections are made by weld deposition or by explosion bonding [6]. Explosion clad plate has only recently become available and its advantages and disadvantages have not yet been fully assessed.

The strength contribution of cladding material to pressure vessel wall is often disregarded in the design and clad is used only to resist corrosion. Typical cladding thicknesses are ranging from 0.125 to 0.1875 in. (3-5 mm) or 10% to 20% of the total thickness of the plate. To assure sound cladding, dye penetrating testing or magnetic crack detection of the cladding are essential requirements [7].

2.3 NON-FERROUS METALS

Metals which do not contain any significant amount of iron in their composition are generally referred to as non-ferrous metals. These materials are used in the chemical industry to eliminate corrosion and undesirable contamination. The most common non-ferrous metals are aluminum, copper, nickel and their alloys.

2.3.1 Aluminum and Aluminum Alloys

The principal reasons for utilizing aluminum in pressure vessel construction are; good ductility at sub-zero temperatures, excellent corrosion resistance and its availability at relatively low cost. These three advantages make aluminum desirable for low temperature applications (storage of liquid methane), the production of hydrogen peroxide, the handling of acids and a number of other duties in chemical and food industries [6].

On the other hand, there are two major properties that restrict the use of aluminum in pressure vessels. These are: (i) low strength and (ii) low melting point. Low strength problems may be overcome to some extent by alloying. In fact, some aluminum alloys have been developed with mechanical properties comparable to those of carbon steels. Low melting point remains a problem, especially in hydrocarbon processing plants, where in the event of a fire, there is a potential danger of melting exposed aluminum parts.

In general, aluminum alloys are classified as heat treatable and non-heat treatable. Of the non-heat treatable, the aluminum alloy of 1.25% manganese, aluminum alloy of 2.75% magnesium and 0.75% manganese, aluminum alloy of 3.5% magnesium and aluminium alloy of 5% magnesium are the most commonly used. Of the heat treatable alloys, the aluminum-magnesium-silicon types are preferred for forged pressure vessel components. Their use in plate form is limited, since their strength is reduced near welded joints due to annealing [9].

In pressure vessels, the application of aluminum alloys is allowed for design temperatures of up to 400°F (205°C).

2.3.2 Copper and Copper Alloys

Commercial copper is available in various grades of purity. It can be classified in two grades: oxygen-bearing copper (electrolytic) and oxygen-free copper (de-oxidized). De-oxidation of copper is necessary in order to avoid porosity and embrittlement in fusion welding [9].

The alloying elements most commonly used with copper are zinc (brasses), tin (bronzes), nickel, silicon, aluminium, cadmium and beryllium. The nomenclature of the copper alloys is rather confusing because of the inconsistent names that have been assigned. For example, an alloy named nickel silver is composed principally of copper, nickel and zinc and does not contain any silver. It has been given this name because of its silvery appearance.

Brasses (Naval Brass, Admiralty Metal, Aluminium Brass, Muntz Metal, etc.) and copper-nickels are used for heat exchanger-tube bundles, because of their ability to resist brackish or salt water.

Bronzes exhibit high resistance for oxidation and corrosion and are stronger than pure copper. Tin bronzes are mainly used for castings, such as valve bodies. Aluminum bronzes are employed for heat exchangers, whenever strong corrosive fluids are handled [6].

2.3.3 Nickel and Nickel Alloys

Nickel and its alloys have a wide application in the chemical and nuclear industries. Nickel is commercially available in two grades, both with impurities content below 1%. The first and more usual grade has a carbon content of about 0.05% and is generally used for handling caustic soda at all temperatures and concentrations, salts, chlorine, fluorine, bromine and molten metals such as sodium and potassium. The second and less common grade has a carbon content of less than 0.02% and it is used for handling molten caustic soda at high temperatures. Nickel is subject to embrittlement and cracking when it is in contact with sulphur and lead [9].

Some of the most important nickel alloys are Monel (66% Ni - 31.5% Cu - 1.5% Fe), Inconel (80% Ni - 15% Cr - 8% Fe), Hastelloy B (28% Mo - 5% Fe), Hastelloy C (16% Mo - 15.5% Cr - 3.5% W - 5.5% Fe), and Incoloy (32% Ni - 22% Cr).

Monel is used for vessels in contact with hydrofluoric acid, caustic soda, salt solutions and salt-water heat exchangers. It is not resistant to oxidizing acids [11].

Inconel is of better corrosion resistance in oxidizing media than Monel or pure nickel. Because of its good creep and thermal shock resistance, Inconel is used as a standard material of construction for nuclear reactors of the boiling type.

The Hastelloys are acid resistant metals. Hastelloy B can easily handle hydrochloric acid, while Hastelloy C is resistant to strong oxidising agents such as ferric chloride.

In spite of the advantages that nickel alloys provide in pressure vessel service, their high cost restricts their application to a few, very special cases. In general, they are replaced by stainless steels, except for temperature above 1110°F (600°C), and for handling highly corrosive fluids.

2.4 OPERATING ENVIRONMENT AND MATERIALS

The selection of a pressure vessel material is dictated primarily by two factors, namely, corrosion resistance and behaviour at the intended temperature application. Corrosion resistance determines partly the ultimate life of the vessel and the degree of product contamination which is likely to occur in service. Temperature of the operating environment affects to a considerable extent, a number of important physical properties of the material.

2.4.1 Corrosion Resistance

Corrosion is the gradual deterioration of a material by a chemical or electrochemical interaction with its environment. This encompasses the deterioration of metals in all types of atmospheres and liquids, and at all temperatures. Corrosion has been the subject of many investigators and consequently, there are many references available in literature, providing valuable information and guidance for the selection of appropriate material for a particular application. For example, corrosion rates for metals exposed to various operating environments may be found in [11].

Acceptable corrosion rates vary greatly according to the industrial application and the severity of the environment. In chemical processes, corrosion rate depends upon the chemical nature of the substances enclosed by a vessel. For example, the purification of crude organic quantities with hydrochloric acid as a by-product, usually results in a highly corrosive environment. Laboratory test data on the comparative corrosion rates of fourteen materials are shown in Fig. 2.1 for the distillation of crude tricresyl phosphate at 550°F (290°C).

In some applications corrosion rate can be as high as 0.05 in. (1.25 mm) per year, and analogous corrosion allowance must be added in order to determine the final wall thickness of the vessel. A common value for standard corrosion allowance is 0.125 in. (3 mm). In some applications the environment is extremely corrosive and no fixed rule for acceptable corrosion allowance can be followed. In such cases, if the only resistant material is a high cost metal, such as platinum for example, it is more economical to use a lower cost metal and accept a shorter vessel life.

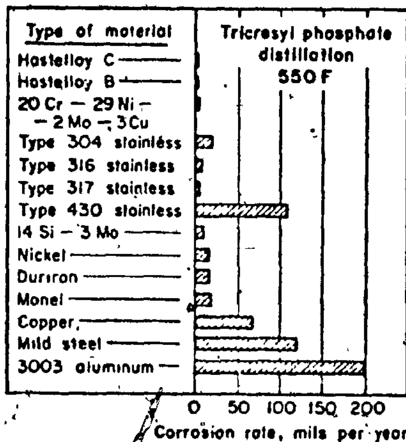

FIG. 2.1 Corrosion Evaluation of Metals for Distilling Tricresyl Phosphate Contaminated with Small Amounts of Hydrochloric Acid [11]

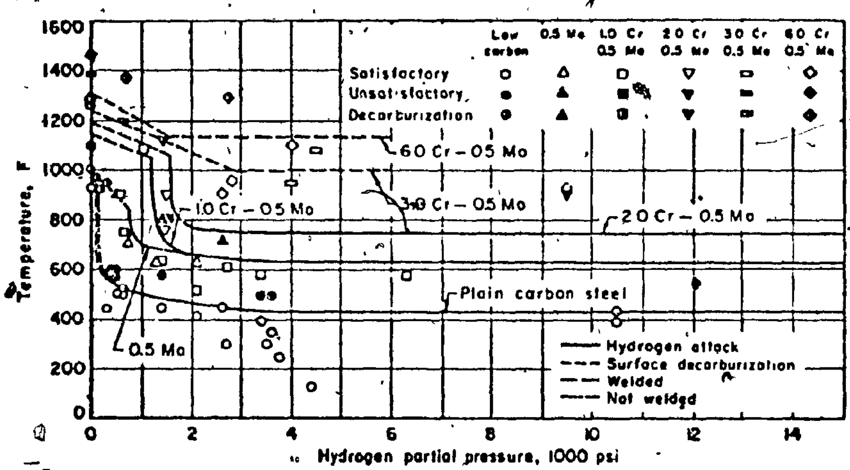

FIG. 2.2 Safe Operating Limits for Carbon and Alloy Steels in Contact With Hydrogen [11]

In petroleum refining processes, hydrogen attacks steels at certain elevated temperatures causing decarburization and fissuring [6]. Carbon steels often are unsuitable for petrochemical application and various degrees of alloying must be employed. Fig. 2.2, also known as 'Nelson's Chart' shows the resistance of carbon and alloy steels to hydrogen attack.

Stress-corrosion cracking is a particularly hazardous form of corrosion, which might result in a catastrophic failure of a vessel. The most common agents promoting such cracking are sodium, magnesium, calcium, zinc or lithium chlorides and moist ethyl chlorides [6]. For austenitic chromium-nickel steel equipment, which may be subject to chloride contamination, stainless-clad steel is more effective than solid stainless steel.

Frequently, carbon steel is coated with both metallic and non-metallic materials to prevent corrosion attacks on the metal. These materials are generally employed as composite steels or linings in the interior surface of the vessel. Non-metallic materials for corrosive environment include brick, ceramic tiles, rubber, wood, glass and concrete. It is apparent that some of these linings limit the operating temperature or pressure or both. Metallic materials of high corrosion resistance are used in a form of a clad plate or a layer of weld deposit in thick sections such as in forgings.

2.4.2 Effects of Temperature on Properties of Metals

Apart from their resistance to corrosive environments, pressure vessel materials must be capable of withstanding a wide range of service temperatures. Temperature effects may be conveniently grouped within three ranges: sub-zero (below 32°F or 0°C), atmospheric (32° to 75°F or 0° to 22°C) and elevated (above 75°F or 22°C).

Vessel materials used for sub-zero temperatures must have good notch ductility. Ductility is a descriptive term related to the ability of the material to be plastically deformed without fracturing in tension [3]. Tensile strength increases with increasing temperature for ductile materials. This property is not so advantageous during the hydrostatic test pressure of a vessel when conducted at room temperature.

Contrary to ductile materials, brittle materials fracture with little or no plastic deformation, and the amount of energy required for fracture is small. Transition temperature is defined as the temperature above which the ductile type of failure occurs. Below the transition temperature a range may exist in which the material has semi-brittle properties. At still lower temperatures the material becomes completely brittle. Below this temperature of complete embrittlement, brittle fracture may occur even though no cracks or notches exist in the material. In the transition range, a crack or notch must exist for brittle fracture to occur [3]. Above the transition temperature, brittle fracture will not occur even if such a notch exists. The transition range is usually determined by conducting Charpy or Izod Impact tests at various temperatures. The procedure for the Charpy impact test for plate steels for vessel construction is described in the ASTM designation A-370-54T [7], and the minimum impact strength is specified. Fig. 2.3 shows the data of low-alloy steel taken by a standard V-notch specimen, as shown in Fig. 2.4.

Grey cast iron may be used at sub-zero temperatures since it is no more brittle under these conditions, than at room temperature [6]. However, grey cast iron has low yield strength. At temperatures down to - 75°F(-60°C), carbon steels are used with higher stresses. Coming further down the temperature scale, at a range of - 60 to - 150°F(-50 to - 100°C), ferritic low alloy steels may be used. Below - 150°F(-100°C) the choice lies between austenitic chromium-nickel steel, 9% nickel steel, aluminium or copper.

In selecting vessel materials for atmospheric temperature, care should be taken to guard against brittle fracture, although there are no mandatory requirements for 'notch brittleness' in pressure vessel codes. A suitable material for atmospheric temperatures is killed or semi-killed fine-grain steels.

Materials at elevated temperatures need to be examined from two points of view, namely, creep strength and metallurgical stability.

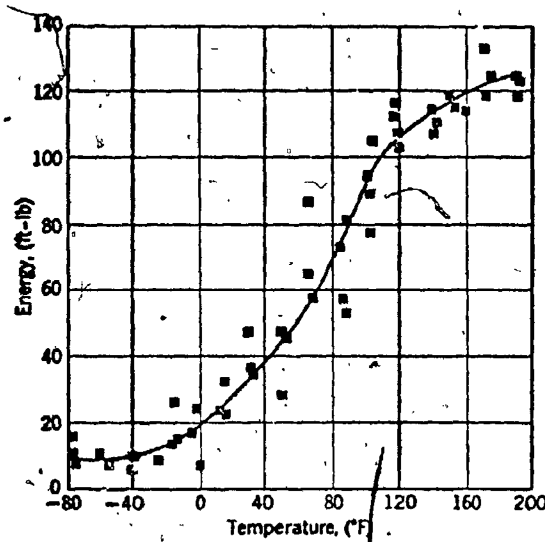

FIG. 2.3 Standard V-Notch Charpy Impact Data for Low-Carbon, Low-Alloy, Hot-Rolled Steel [3]

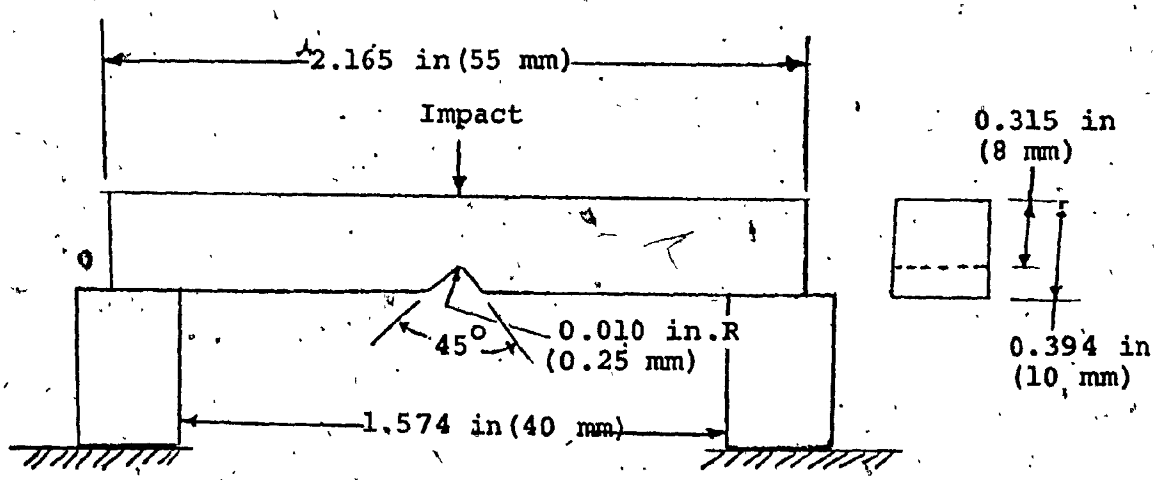

FIG. 2.4 Standard Charpy V-Notch Specimen [7]

Creep is the continuous deformation of the material under constant load. Creep properties may be improved if the composition is such that precipitation of carbides takes place during the creep process. The effect of certain elements on the creep strength is shown in Fig. 2.5. From this figure one can see that the creep strength of ferritic steels is significantly enchanced by small additions of molybdenum [10].

The strength of materials is also dependent on their microstructure. Transformations take place at high temperatures resulting in different products such as ferrite, pearlite, bainite, martenite, etc. Extensive description of these transformed products and their effect on strength can be found in standard metallurgical books [10]. Carbon, carbon-manganese and carbon-molybdenum steels with low carbon content used in pressure vessel construction are normally pearlitic.

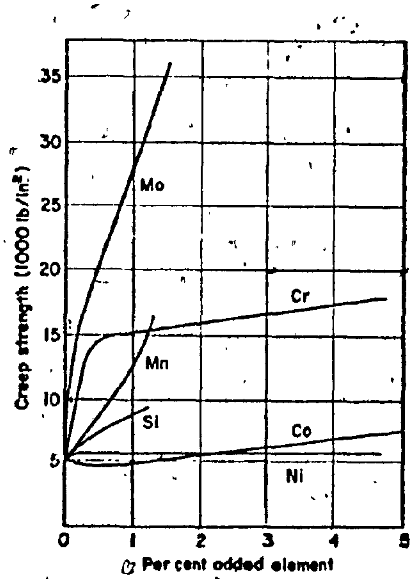

FIG. 2.5 Effect of Certain Elements on Creep Strength of Iron at 800 F - 0.0001% per Hour [10]

Temper embrittlement, spheroidisation, graphitisation, carbide precipitation or sigma phase formation, are some of the metallurgical changes that weaken the material properties and must be avoided. Secondary hardening, that is hardening due to carbide precipitation at elevated temperatures, has been known to occur but can be controlled by post-weld heat treatment at a proper temperature. A material with good metallurgical stability is Incoloy 800 (32% Ni - 22% Cr) which exhibits high strength at elevated temperatures [11]. Semi-killed carbon steel, silicon-killed steel, 1% chromium 0.5% molybdenum steel and austenitic chromium-nickel steel are some of the recommended materials for vessels at elevated temperatures.

CHAPTER III

STRESSES IN PRESSURE VESSELS

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STRESSES IN PRESSURE VESSELS

3.1 GENERAL SHELL THEORIES

The most common shell theories are those based on linear elasticity concepts. These theories predict to an acceptable level of accuracy, stresses and deflections of shells exhibiting linear elastic deformation. The basic assumption of all such theories is that the resulting deformations follow Hooke's law and that the equilibrium conditions are satisfied before and after deformations have occurred.

The non-linear theories of elasticity apply generally to shells which manifest large deformations. These theories are often required when dealing with shallow shells, highly elastic membranes and shells subjected to buckling loads. In most cases, non-linear theories result in complicated equations and for this reason they are not widely used.

The theory of small deflections of thin elastic shells, is based upon the equations of the two-dimensional linear elasticity. Basic relations can be developed from either statical equilibrium or from purely geometrical considerations. The geometry of a shell is entirely defined by its reference surface, its thickness and its boundary [12]. Of these, the reference surface, that is, the locus of points equidistant from the bounded surfaces, is the most significant since it defines the shape of the shell. In the analysis of such shells, the governing equations of the three-dimensional theory of elasticity involving three independent space variables are reduced to a new system with only two space variables. These two variables are more conveniently taken as coordinates on the reference surface of the shell.

In general, linear shell theories can be classified into four essential categories [13].

(1) First-order approximation shell theory

(2) Second-order approximation shell theory

(3) Specialized theories of shells

(4) Membrane shell theory

The order of a particular approximate theory is established by the order of terms in the thickness coordinate that is retained in the strain and constitutive equations. The introduction of certain assumptions permits the evaluation of stress-resultant equations, thereby rendering approximate relationships between force and deformations.

In the case of thin elastic shells, the simplified bending theories are generally based on Love's first and second approximation shell theories. Love was the first investigator to present a successful approximation theory based on classical elasticity in 1888 [12].

Special shell theories have been developed to include the shallow-shell theory, shear deformation and Geckeler's approximation for symmetrically loaded shells [13]. These three approaches can accommodate force and moment fields.

The shell theories stated above are generally referred to as 'bending' theories because their development included the consideration of the flexural behaviour of the shells. If, in the study of equilibrium of a shell all moment expressions were neglected, the resulting analysis would be the membrane theory of shells.

A shell can be considered to act as a membrane if it offers no resistance to bending and if only a momentless state of stress exists. In many practical designs, the resultant forces do, in fact, produce minor bending moments which can be neglected. The momentless state of stress condition is a desirable feature in the analysis of shell structures because it results in simple design formulas incorporating a high factor of safety.

The study of the membrane theory is considerably simpler than that of the bending theories and for this reason, historically preceded the latter. The first contributions to the membrane theory were furnished by Lamé and Clapeyron in the early nineteenth century [13]. A detailed discussion of all theories dealing with thin elastic shells, can be found in [12,13].

3.2 MEMBRANE STRESSES IN SHELLS UNDER INTERNAL PRESSURE

The membrane theory can be employed to find stresses in all types of shells of revolution. A shell of revolution is obtained by the rotation of a plane curve about an axis lying in the plane of the curve. This curve is called a meridian and its plane is the meridian plane. The intersections of the surfaces with planes perpendicular to the axis of rotation are parallel circles and are called parallels. The two coordinates of a shell of revolution are taken along the meridian and parallel and are identified by subscripts 1 and 2, respectively.

The membrane stresses in shells of revolution can be calculated using the equations of statics, provided that the shells are loaded in a rotationally symmetrical manner. The only load that will be considered in the following evaluation of stresses is internal pressure. For external or concentrated loads, the membrane theory is not applicable.

In the shell of Fig.3.1(a), if an element abce is cut by two meridional sections, ab and ec and by two parallel sections ae and bc normal to these meridians, it is seen that only normal stresses are present and symmetry exists [14]. The symbols used in this element are defined as follows:

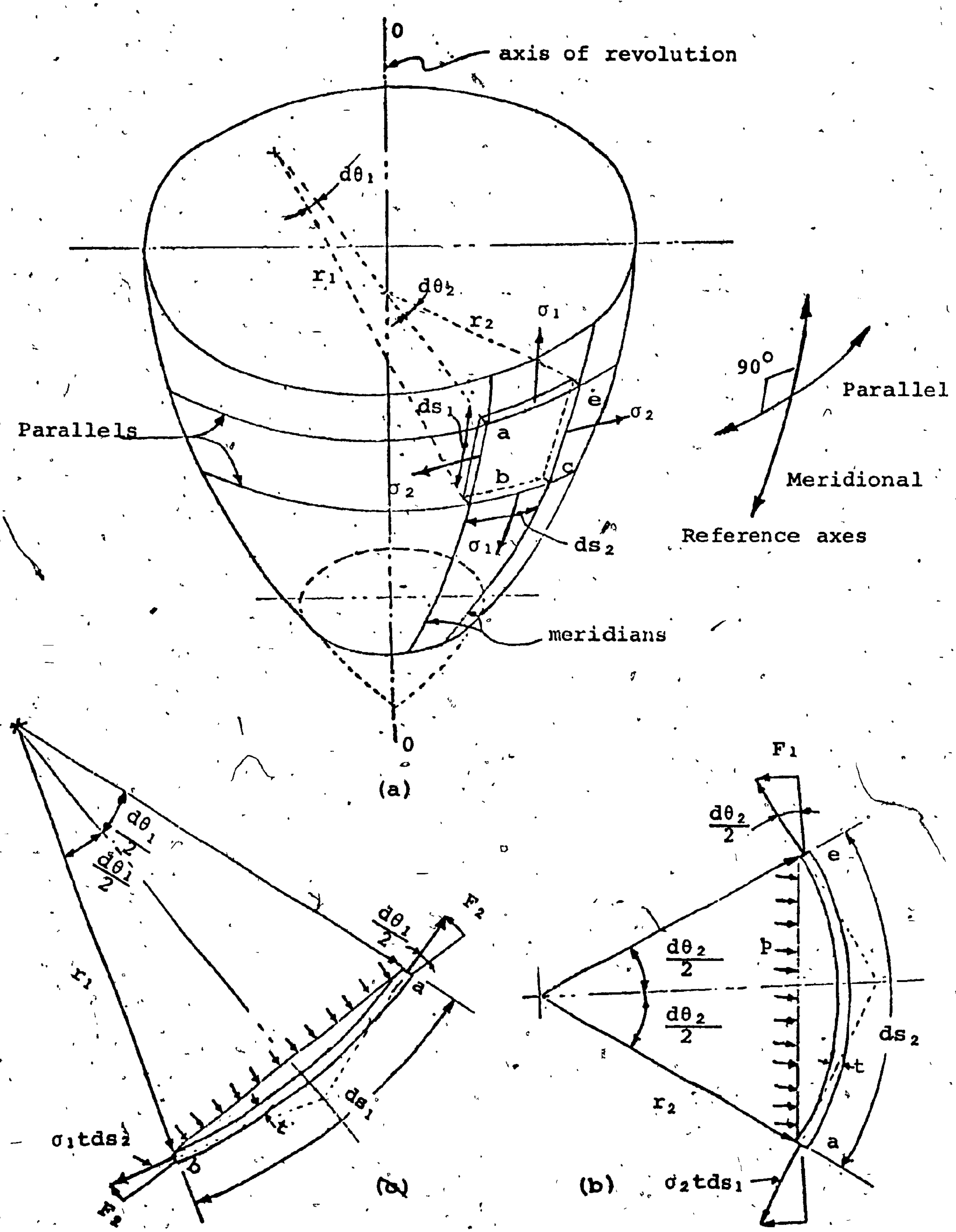


FIG.3.1 Membrane Stress in Shells of Revolution [14]

σ_1 = longitudinal or meridional stress

σ_2 = hoop stress or stress along a parallel circle

t = thickness of the shell

ds_1 = element arc in the meriodional direction

ds_2 = element arc in the tangential direction

r_1 = meridional radius of curvature

r_2 = tangential radius of curvature

p = internal pressure

Referring to Fig. 3.1, the forces acting on the sides of the element are $\sigma_1 t ds_2$ and $\sigma_2 t ds_1$. In Fig. 3.1(b), the force $\sigma_2 t ds_1$ has a component in a direction normal to the element

$$2F_1 = 2\sigma_2 t ds_1 \sin(\frac{d\theta_2}{2}) \quad (3.1)$$

and similarly in Fig. 3.1(c), the force $\sigma_1 t ds_2$ has a component in a direction normal to the element

$$2F_2 = 2\sigma_1 t ds_2 \sin(\frac{d\theta_1}{2}) \quad (3.2)$$

The total pressure acting on the element is

$$P = p[2r_1 \sin(\frac{d\theta_1}{2})][2r_2 \sin(\frac{d\theta_2}{2})] \quad (3.3)$$

which is in equilibrium with the sum of the normal membrane component forces, hence,

$$2\sigma_2 t ds_1 \sin(\frac{d\theta_2}{2}) + 2\sigma_1 t ds_2 \sin(\frac{d\theta_1}{2}) =$$

$$= p[2r_1 \sin(\frac{d\theta_1}{2})][2r_2 \sin(\frac{d\theta_2}{2})] \quad (3.4)$$

For small angles

$$\sin\left(\frac{d\theta_1}{2}\right) = \frac{ds_1}{2r_1}, \quad \sin\left(\frac{d\theta_2}{2}\right) = \frac{ds_2}{2r_2}$$

and, substituting in Eq. (3.4), gives

$$p\left[2r_1 \frac{ds_1}{2r_1}\right]\left[2r_2 \frac{ds_2}{2r_2}\right] = 2\sigma_1 t ds_2 \frac{ds_1}{2r_1} + 2\sigma_2 t ds_1 \frac{ds_2}{2r_2}$$

or

$$p ds_1 ds_2 = t ds_1 ds_2 \left(\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2}\right)$$

which yields

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t} \tag{3.5}$$

The signs of the radii of curvature in Eq. (3.5) are both positive since they point toward the center of the shell. If the radius points away from the center of the shell, it is negative. Also, worth mentioning is that the internal pressure need not be constant along the axis of the vessel, as it would be the case for shells containing liquids. However, it is assumed that the pressure acting on the element under consideration is constant.

Applications of Eq. (3.5) for simple geometric shapes with constant radii of curvature are demonstrated in the following section.

3.2.1 Cylindrical Shells

In a cylindrical shell, the radius which defines the curvature of the parallels (tangential radius) is equal to that of the cylinder, and the radius along the axis of revolution (longitudinal radius) is infinitely large, that is, $r_2 = r$, and $r_1 = \infty$. Therefore, from Eq. (3.5) the hoop stress σ_2 of a cylindrical section under internal pressure p is

$$\frac{\sigma_1}{\infty} + \frac{\sigma_2}{r} = \frac{p}{t}$$

or

$$\sigma_2 = \frac{pr}{t} \tag{3.6}$$

The longitudinal stress can be calculated by equating the total force exerted by pressure against the end of the cylinder to the longitudinal forces acting on a transverse section of the cylinder, that is

$$2\pi r \sigma_1 t = \pi r^2 p$$

or

$$\sigma_1 = \frac{pr}{2t} \tag{3.7}$$

3.2.2 Spherical Shells

In the case of a spherical shell, the longitudinal and tangential radii are equal. That is, $r_1 = r_2 = r$, and in view of the symmetry of the two coordinate axes, it follows that $\sigma_1 = \sigma_2 = \sigma$.

Thus, Eq. (3.5) reduces to

$$\sigma = \frac{pr}{2t} \quad (3.8)$$

3.2.3 Conical Shells

In a conical shell, it is seen from Fig. 3.2 that $r_1 = \infty$ and $r_2 = r/\cos\alpha$. Thus, substituting r_2 in Eq. (3.5), the hoop stress is

$$\sigma_2 = \frac{pr}{t\cos\alpha} \quad (3.9)$$

In Eq. (3.9), as $\alpha \to 0^\circ$ the hoop stress approaches that of a cylinder and as $\alpha \to 90^\circ$ the stress becomes infinitely large and the cone flattens out into a plate [14].

The longitudinal stress is found by equating the axial component of the force in the shell wall, to the total force exerted by pressure on a plane perpendicular to the axis of revolution. Therefore

$$\sigma_1 t 2\pi r \cos\alpha = p\pi r^2$$

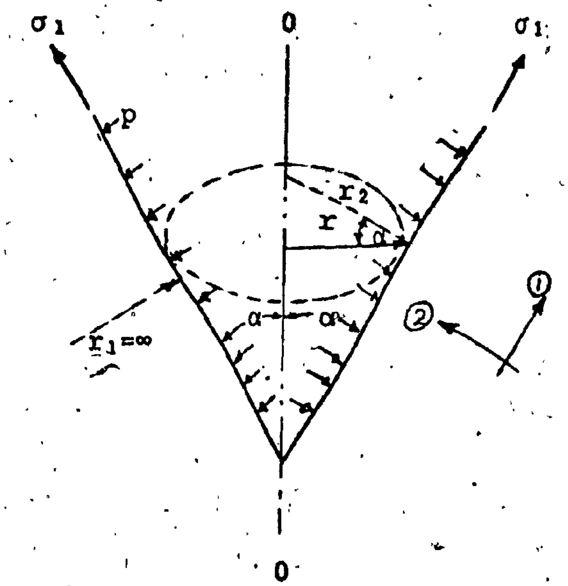


FIG. 3.2 Conical Shell Under Internal Pressure

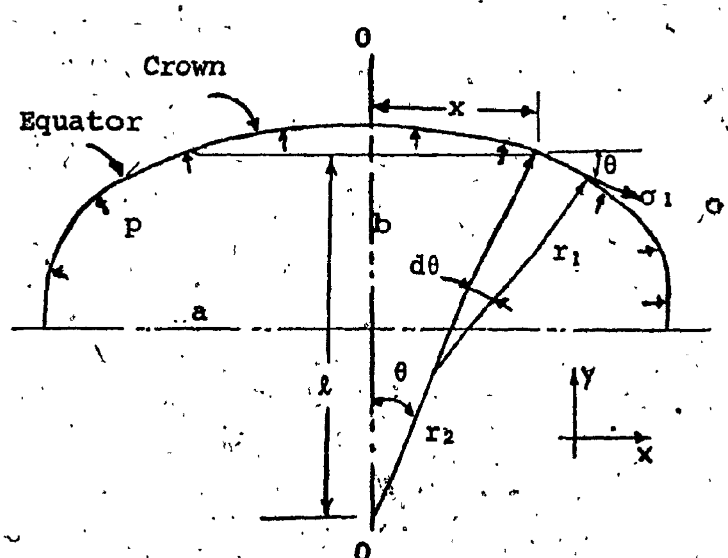


FIG. 3.3 Ellipsoidal Shell Under Internal Pressure

or

$$\sigma_1 = \frac{pr}{2t\cos\alpha} \qquad (3.10)$$

3.2.4 Ellipsoidal Shells

In the case of an ellipsoidal shell, Fig. 3.3, the radius of curvature varies from point to point and is given by

$$\rho = \frac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2} \qquad (3.11)$$

The equation of an ellipse is

$$y = \pm \frac{b}{a}\sqrt{a^2-x^2} \qquad (3.12)$$

Differentiating twice Eq. (3.12) gives

$$\frac{dy}{dx} = \frac{-bx}{a\sqrt{a^2-x^2}} = -\frac{b^2x}{a^2y} \qquad (3.13)$$

$$\frac{d^2y}{dx^2} = \frac{-ba^2}{a(a^2-x^2)^{3/2}} = \frac{-b^4}{a^2y^3} \qquad (3.14)$$

Substituting Eq. (3.13) and (3.14) into Eq. (3.11), yields

$$r_1 = |\rho| = \frac{[1+(b^2x/a^2y)^2]^{3/2}}{b^4/a^2y^3} =$$

$$= \frac{[a^4y^2+b^4x^2]^{3/2}}{a^4b^4} \qquad (3.15)$$

The radius r_2 in the hoop direction, is the length of the normal to the axis of revolution 0-0 [15], hence

$$\tan\theta = \frac{dy}{dx} = \frac{bx}{a\sqrt{a^2-x^2}} = \frac{x}{\ell} \tag{3.16}$$

or

$$\ell = \frac{a}{b}\sqrt{a^2-x^2} \tag{3.17}$$

and from the right triangle relation it follows that

$$r_2 = \sqrt{\ell^2+x^2} = \frac{1}{b^2}\sqrt{a^4y^2+b^4x^2}$$

and

$$r_1 = r_2^3\frac{b^2}{a^4} \tag{3.18}$$

The longitudinal stress can be calculated as in the case of a cone by considering the equilibrium of the portion of the ellipse above the circle of radius x [15]. Thus

$$2\pi xt\sigma_1 \sin\theta = x^2\pi p, \quad \sin\theta = \frac{x}{r_2}$$

or

$$\sigma_1 = \frac{px}{2t(x/r_2)}$$

$$\sigma_1 = \frac{pr_2}{2t} \tag{3.19}$$

The hoop stress equation is derived from Eq. (3.5), that is

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}$$

$$\sigma_2 = \frac{pr_2}{t} - \frac{\sigma_1 r_2}{r_1} \quad \text{and} \quad \sigma_1 = \frac{pr_2}{2t}$$

$$\sigma_2 = \frac{p}{t}\,(r_2 - \frac{r_2^2}{2r_1}) \tag{3.20}$$

At the crown $r_1 = r_2 = \frac{a^2}{b}$ and from Eq. (3.19)

$$\sigma_1 = \sigma_2 = \frac{pa^2}{2bt} \tag{3.21}$$

At the equator $r_2 = a$ and $r_1 = \frac{b^2}{a}$. Also from Eq. (3.19)

$$\sigma_1 = \frac{pa}{2t}$$

and

$$\sigma_2 = \frac{p}{t}\,(a - \frac{a^3}{2b^2}) = \frac{pa}{t}\,(1 - \frac{a^2}{2b^2}) \tag{3.22}$$

The relation of the radius of the shell to the depth of the ellipsoid determines the state of stresses as shown in Fig. 3.4. If $a/b = 1.42$ the value of σ_2 goes from maximum tension at the center of the crown to zero at the boundary of the major axis [16]. If $a/b > 1.42$, σ_2 has its

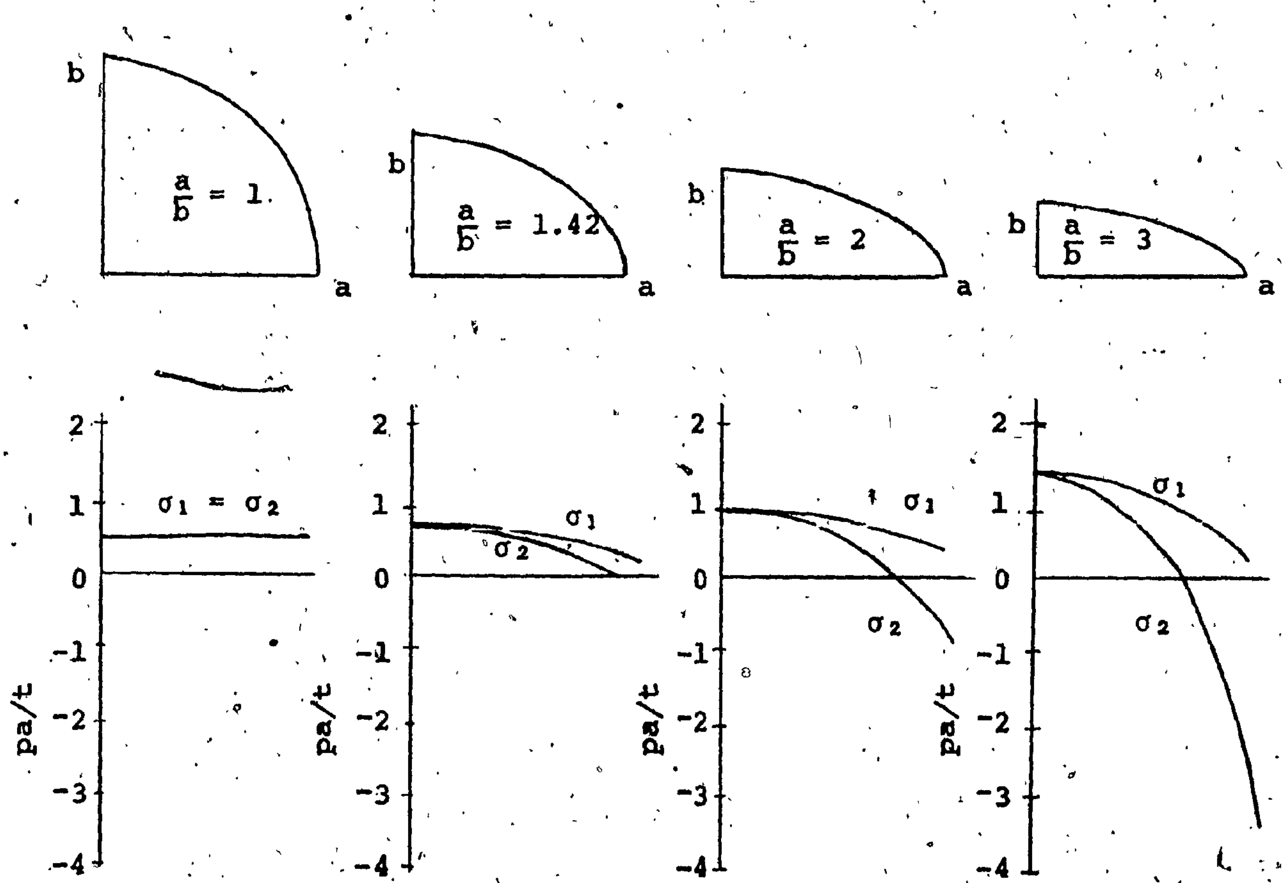


FIG. 3.4 Ratios of Stresses in Ellipsoidal Shell to Stress in Cylinder With Radius Equal to One-Half the Major Axis a, Versus Distance from the Center to the Boundary Along the Major Axis a [16]

maximum value at the boundary and becomes negative at the equator. The meridional stress remains tensile throughout the ellipsoidal for all a/b ratios, being a maximum at the crown and diminishing in value to a minimum at the equator.

3.3 DISCONTINUITY STRESSES

In addition to the principal (membrane) or primary stresses, discontinuity or secondary stresses exist. Principal stresses are produced by internal or external pressure and remain only as long as the pressure is maintained. Other loads also giving rise to primary stresses are wind, snow, earthquakes, dead weight, etc. These may result in either tension, compression or bending and must be considered together with the stresses generated by the pressure fields in determining the total primary stresses.

Secondary stresses, on the other hand, are those localized stresses developed as a result of geometrical discontinuities and physical constraints acting in a section of a structure. For example, discontinuity stresses may occur at the junction of a cylindrical vessel and its closure head resulting from the differential growth or dilation of these parts under pressure. Localized secondary stresses are also developed around nozzles or support attachments and their magnitude depends primarily on external loads. Furthermore, secondary stresses include all stresses produced by thermal gradients (thermal stresses) within the structure. Secondary stresses produce local yielding or minor distortion in the vessel elements. Thus, high local stresses can be accomodated without failure in a limited number of stress cycles. However, under repetitive loading conditions, secondary stresses affect material fatigue life and can lead to failure.

In the design of conventional pressure vessels, secondary stresses can be disregarded. This is the case in Section I and Section VIII, Division 1, of the ASME Code, where discontinuity stress analysis is not required and conservative design stress values are used. However, in nuclear equipment design where stringent rules apply, secondary stresses cannot be neglected. In Section III, Class 1 and 2, and Section VIII, Division 2 of the ASME Code, all secondary stresses must be analyzed and kept within certain prescribed limits.

3.3.1 Stresses at the Head-Shell Junction

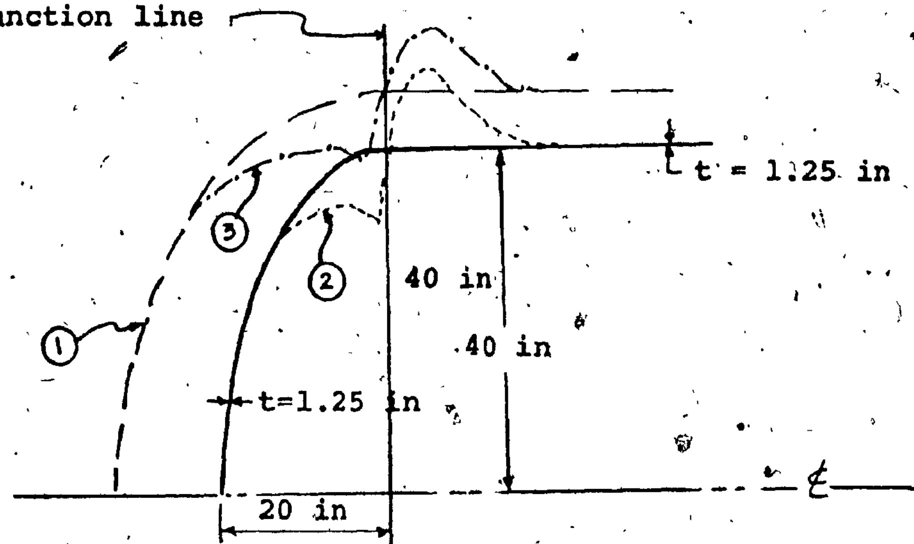
The discontinuity in shape existing at the junction of a cylindrical vessel and its closure results in localized stress concentrations. In conventional service, where usually the magnitude of these stresses is low, they are neglected. However, for adequate evaluation of a design a knowledge of the magnitude of these stresses is essential.

The nature of the stress concentrations is complex in that bending moments, shear and stress reversals must be considered in addition to the membrane stresses resulting from internal pressure. The evaluation of discontinuity stresses at or near the junction, has been the subject of many investigators [17] and is beyond the scope of this report.

The great majority of cylindrical vessels have either elliptical or torispherical heads. Although the hemispherical dished head is stronger than either the elliptical or torispherical one, it is not so widely used because of the excessive forming required in its fabrication. In general, this results in a higher fabricating cost and a more limited range of available sizes. The principal advantage of dished heads over flat cover plates or cones as closures, is the presence of lower discontinuity stress at the junction of a cylindrical shell with its closure [18].

W.M. Coates [17] mathematically investigated the state of stresses at the junction of a dished head and a cylindrical shell. He analyzed a longitudinal strip of the shell in the neighborhood of the junction being bent inward by the deformation of the head, and considered this strip to act like a beam on an elastic foundation. Assuming that the deflection of the beam is proportional to the applied force, Coates developed relations for the discontinuity stresses.

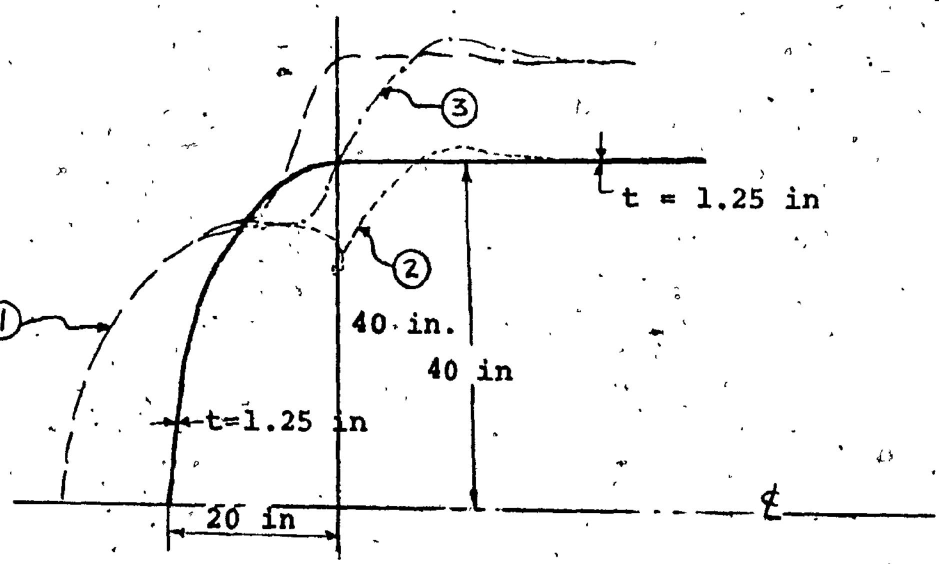
To illustrate the method, Coates chose a cylindrical steel vessel with an elliptical dished head, having a major-to-minor axis ratio of 2:1 and a shell diameter of 80 in. The vessel was assumed to have a uniform thickness of 1.25 in and was to operate under an internal pressure of 100 psig. The resulting stresses near the head-shell junction are shown in Fig. 3.5. The membrane and discontinuity stresses are indicated



① — — — Membrane Stress (Meridinal

② - - - - - - - - - Discontinuity Stress (bending)

③ —·—·— Resultant Meridional Stress



① — — — Membrane Stress (hoop)

② - - - - - - - - - Discontinuity Stress

③ —·—·— Resultant Hoop Stresses

FIG. 3.5 Stress at and Near the Head-Shell Junction [17] separately. The resultant stresses at and near the junction are obtained by the superposition of these two.

The discontinuity of the hoop stress in cylindrical shells and their closure, loaded by internal pressure, may be avoided by choosing a particular meridian configuration. Flügge has demonstrated [19] that Cassinian curves are particularly suitable in this application. These curves are illustrated in Fig. 3.6 and their equation is

$$(r^2+n^2z^2)^2 + 2a^2(r^2-n^2z^2) = 3a^4$$

where n is a number greater than one ($n > 1$), r and z are coördinate lines and a is equal to the maximum r.

Fig. 3.7 gives an example of the distribution of the stress resultants in an elliptical dished closure. It shows no discontinuity in the hoop stress. There is a small zone in which the meridional stress is negative. This may be avoided by choosing $n < 1.9$. If n is chosen much greater than 2, the compressive zone is wider and the maximum compressive stress is higher [19].

3.3.2 Local Stresses Due to External Loading

In addition to withstanding pressure at all operating and test conditions, a vessel must be capable of carrying a number of other loads, which are applied through attachments such as discrete support brackets, lifting lugs and nozzles.

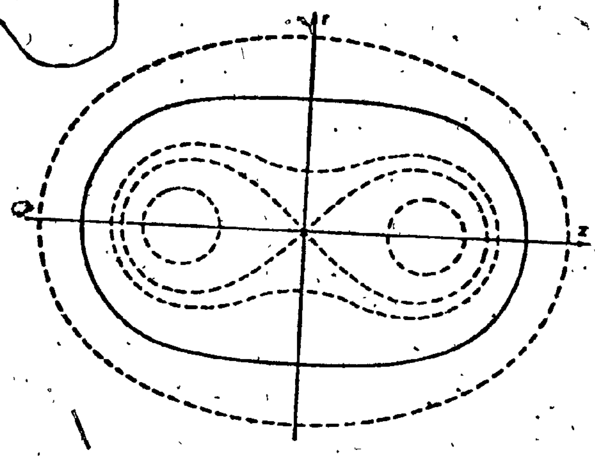


FIG. 3.6 Cassini's Curves

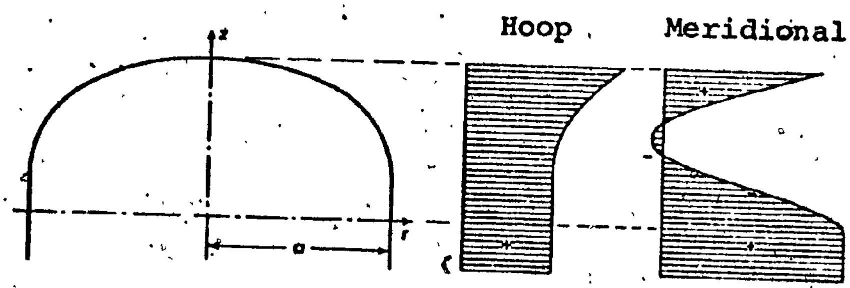


FIG. 3.7 Elliptical Head Without Discontinuity in the Hoop Stress [19]

In the case of supports and lifting lugs, the loads will arise mainly due to dead weight, whereas nozzle attachments transmit a load imposed through piping systems. All such loads are referred to as local loads because any significant effects on the main shell are usually confined to areas in close proximity to the attachment.

Bijlaard considered a number of different external load combinations applied on shells over a rectangular or circular area [11]. The analysis was published between 1954 and 1959 and covered radial loads and bending moments on local areas of cylindrical and spherical shells.

Bijlaard's results are presented in terms of non-dimensional quantities such as

$$\frac{M_x}{P}, \quad \frac{N_x}{(P/r)}, \quad \frac{N_\theta}{(M/r^2\beta)}, \quad \frac{M_x}{(M/r\beta)}, \quad \frac{\text{Rotation}}{(M/r^3\beta^2E)}$$

where

M_x = bending moment in shell wall in radial direction, in-lb (N-m)

P = total distributed radial load, lb (N)

N_x = membrane forces in shell wall in radial direction, lb/in (N/m)

N_θ = membrane force at an angle θ around attachment, lb (N)

M = concentrated external overturning moment, in-lb (N-m)

r = radius of the shell, in (mm)

E = modulus of elasticity, psi (MPa)

β = attachment parameter, (c/r)

c = loading parameter

γ = shell parameter (r/t)

t = shell thickness, in (mm)

These non-dimensional quantities can be used to find deflections, direct and bending stresses for a given type of loading in the circumferential and longitudinal direction. The β and γ parameters are essential in defining the geometry of the shell and the attachments. These results have also been plotted by the use of a digital computer [20]. To facilitate the computation process, stress components at specific points have been tabulated using a fixed sign convention. Fig. A-4 is a typical example of this presentation taken from [20].

CHAPTER IV

DESIGN OF PRESSURE VESSELS

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DESIGN OF PRESSURE VESSELS

4.1 INTERNAL PRESSURE

4.1.1 Design/Equations

The basis for the design of cylindrical, spherical, ellipsoidal and other types of pressure vessels under internal pressure is a modified membrane theory, which allows for stress variations across the wall thickness.

The membrane stress equation for the hoop stresses in a cylindrical shell was shown previously to be

$$S = \frac{PR}{t} \qquad (4.1)$$

$$S = \frac{PR}{1000(t)} \qquad (4.1^*)$$

for a wall thickness which is small compared to the radius. For a cylindrical shell with a significant thickness to radius ratio (t/R), the following relationship holds [21].

---

*Asterisk indicates that the equation is in SI Units.

$$S = \frac{P\,R_i^2}{R_0^2 - R_i^2}\left(1 + \frac{R_0^2}{R^2}\right) \qquad (4.2)$$

$$S = \frac{PR_i^2}{1000(R_0^2 - R_i^2)}\left(1 + \frac{R_0^2}{R^2}\right) \qquad (4.2^*)$$

where R is the radial distance, R_i and R_0 are inside and outside radii respectively, and P is the internal pressure. Replacing the outside-to-inside radius ratio by K

$$K = \frac{R_0}{R_i} \qquad (4.3)$$

and substituting in Eq. (4.1) with the thickness expressed in terms of the radii as $t = R_0 - R_i$, and with $R = R_i$, then

$$S = \frac{P}{K - 1} \qquad (4.4)$$

$$S = \frac{P}{1000(K - 1)} \qquad (4.4^*)$$

Similarly, Eq. (4.2) becomes

$$S = \frac{R_i^2 P}{R_i^2 (R_o^2/R_i^2 - 1)} \left(1 + \frac{R_o^2}{R^2}\right)$$

$$S = \frac{P}{K - 1} \left(1 + \frac{R_o^2}{R^2}\right) \qquad (4.5)$$

Inspection of Eq. (4.5) shows that maximum stress value occurs when $R = R_i$, hence

$$S = \frac{P}{K^2 - 1} \left(1 + \frac{R_o^2}{R_i^2}\right)$$

$$S = \frac{K^2 + 1}{K^2 - 1} (P) \qquad (4.6)$$

The above stress formula, known as the 'Lamé Equation' is used in connection with the design of thick-walled cylindrical pressure vessels [15, 18]. For K values approaching unity $K^2 + 1 \to K + 1$ and thus Eq. (4.6) becomes

$$S = P \frac{K^2 + 1}{(K-1)(K+1)}$$

or

$$S \approx \frac{P}{K - 1}$$

In pressure vessels, Eq. (4.5) is empirically modified by introducing a constant of 0.6, hence

$$\frac{S}{P} = \frac{1}{K - 1} + 0.6 \qquad (4.7)$$

The above equation shows a good agreement with the Lamé formula for values of $K < 3.0$ [15].

Eq. (4.7), rewritten in terms of inside radius and wall thickness, yields

$$S = P\left[\frac{1}{(R_i+t)/R_i - 1} + 0.6\right]$$

or

$$S = \frac{PR_i}{t} + 0.6P$$

and a design formula for thickness may be obtained, thus

$$t = \frac{PR_i}{S - 0.6P} \qquad (4.8)$$

$$t = \frac{PR_i}{1000(S) - 0.6P} \qquad (4.8^*)$$

Eq. (4.8) is used in the ASME Code to determine the thickness of a cylindrical vessel under internal pressure P.

Eq. (4.7) can also be expressed in terms of the outside radius, thus

$$S = P\left[\frac{1}{R_0/(R_0-t) - 1} + 0.6\right]$$

$$S = \frac{P(R_0-t)}{t} + 0.6P$$

or

$$t = \frac{PR_0}{S + 0.4P} \qquad (4.9)$$

$$t = \frac{PR_0}{1000(S) + 0.4P} \qquad (4.9^*)$$

In the same way other empirical formulas have been derived for other shapes of shells which are presented in the following section.

4.1.2 Thickness of Shells

The thickness of a cylindrical or spherical shell can be computed by the following formulas where

t = minimum required thickness of shell plates, exclusive of corrosion allowance in (mm)

P = design pressure, psi (kPa)

R = inside radius of the shell, before corrosion allowance is added, in (mm)

S = maximum allowable stress value, psi (MPa)

E = joint efficiency of welds. (See Fig. A-2)

Cylindrical shells:

In accordance with ref.[22], the minimum thickness or maximum allowable working pressure of cylindrical shells shall be the greater thickness or lesser pressure as given by (1) or (2) below:

(1) When the thickness does not exceed one-half of the inside radius, or P does not exceed 0.385SE, Eq.(4.8) derived above, applies

$$t = \frac{PR}{SE - 0.6P} \quad \text{or} \quad P = \frac{SEt}{R + 0.6t} \qquad (4.10a)$$

$$t = \frac{PR}{(1000)SE-0.6P} \quad \text{or} \quad P = \frac{(1000)SEt}{R + 0.6t} \qquad (4.10a^*)$$

E is the joint efficiency of the longitudinal welds.

(2) When the thickness does not exceed one-half of the inside radius, or P does not exceed 1.25 SE, the following formula applies

$$t = \frac{PR}{2SE + 0.4P} \quad \text{or} \quad P = \frac{2SEt}{R - 0.4t} \qquad (4.10b)$$

$$t = \frac{PR}{(1000)2SE+0.4P} \quad P = \frac{(1000)SEt}{R - 0.4t} \qquad (4.10b^*)$$

E is the joint efficiency of circumferential welds.

Spherical shells:

When the thickness of the shell does not exceed 0.356R, or P does not exceed 0.665 SE, the following formulas apply.

$$t = \frac{PR}{2SE - 0.2P} \quad \text{or} \quad P = \frac{2SEt}{R + 0.2t} \qquad (4.11)$$

$$t = \frac{PR}{(1000)2SE\ 0.4P} \quad \text{or} \quad P = \frac{(1000)SEt}{R + 0.4t} \qquad (4.11^*)$$

When dealing with corrosive materials, the final thickness of a shell must be more than that determined by the above formulas. This extra thickness is known as corrosion allowance and it usually ranges from 0.062 in. to 0.25 in.

4.2 EXTERNAL PRESSURE

Often, pressure vessels are required to operate under partial or full vacuum or under other forms of external pressure. Typical examples are, vacuum condensers for evaporators and distillation columns, vacuum columns, diving chambers, etc.

The equations of the membrane theory do not hold if pressure is applied to the convex side of a shell of revolution. Instead of the tensile hoop or meridional stress, the ability of the shell to withstand local buckling becomes the governing factor in assuring structural integrity. The collapsing strength of externally pressurized vessels may be increased by the use of uniformly spaced, internal or external circumferential stiffening rings. From the standpoint of elastic stability, such stiffeners have the effect of subdividing the length of the shell into subsections equal in length to the center-to-center spacing of the stiffeners.

4.2.1 Allowable External Pressure on Shells

A cylindrical shell under external pressure tends to deform inward as a result of external radial pressure. Examining the shell section after it has been deformed,a system of equations based on elastic stability theory can be formulated,which after extensive manipulations yields the following Equation [23]

$$P = \frac{3EI}{R^3} = \frac{24EI}{D_0^3} \qquad (4.12)$$

$$P = \frac{(1000)(24)EI}{D_0^3} \qquad (4.12^*)$$

Eq. (4.12) gives the theoretical buckling pressure P per unit circumferential length and width of circumference of a shell with an original radius R. This equation is not valid when the shell section is part of a cylindrical vessel, where any adjacent attachment may act as a restraint to the longitudinal deformation. To account for this restraint Eq. (4.12) is divided by $(1 - \nu^2)$, and substituting the moment of inertia for a rectangular strip with a unit width (b = 1), I = $bt^3/12$, gives

$$P = \frac{2E}{1-\nu^2}\left(\frac{t}{D}\right)^3 \qquad (4.13)$$

For most pressure vessel materials Poisson's ratio ν = 0.3 and Eq. (4.13) reduces to

$$P = 2.2E\left(\frac{t}{D}\right)^3 \qquad (4.14)$$

$$P = (1000)(2.2)E\left(\frac{t}{D}\right)^3 \qquad (4.14^*)$$

Eq.(4.14) gives the theoretical critical pressure assuming that the vessel is perfectly round, a condition rarely found in actual cases.

Experimentally, it was found that ordinary tubes and pipes buckle at external pressures of about 27% less than the theoretically predicted pressure. For a maximum allowable buckling pressure, a factor of safety of four (4) is applied to Eq. (4.14) to obtain a safe design pressure

$$P_{all} = 0.55\,E\left(\frac{t}{D}\right)^3 \qquad (4.15)$$

$$P_{all} = (1000)(0.55)E\left(\frac{t}{D}\right)^3 \qquad (4.15^*)$$

Eq. (4.15) does not apply to relatively short cylinders or long cylinders with stiffening rings spaced at less than the critical length. Critical length is that length of a cylindrical shell beyond which restraints have no stiffening effect. An expression for the critical length was developed by Southwell for a cylinder, given by [23]

$$L_c = 1.11\ D\sqrt{D/t} \qquad (4.16)$$

Cylinders shorter than the critical length or for cylindrical vessels with circumferential stiffening rings, Eq. (4.15) can be modified to include the stiffening effect of short distance against buckling. Thus, with a factor of safety of four (4), Eq. (4.15) results in

$$P_{all.} = \frac{KE}{4}\left(\frac{t}{D}\right)^3 \qquad (4.17)$$

$$P_{all.} = \frac{(1000)KE}{4}\left(\frac{t}{D}\right)^3 \qquad (4.17^*)$$

where K is a non-dimensional coefficient determined from the vessel parameters D/t and L_s/D where L_s is the length between circumferential stiffeners.

Substituting Eq. (4.17) into the membrane formula, the circumferential compressive stress at which collapse occurs is found as follows:

$$S = \frac{PR}{t} = \frac{R}{t}\,\frac{KE}{4}\left(\frac{t}{D}\right)^3$$

$$S = \frac{KE}{2} \left(\frac{t}{D}\right)^2 \tag{4.18}$$

Eq.(4.18) can be divided by the modulus of elasticity E, to be expressed in terms of the strain ε, that is

$$\varepsilon = \frac{S}{E} = \frac{K}{2} \left(\frac{t}{D}\right)^2 \tag{4.19}$$

Eq.(4.19) has been plotted as shown in Fig. A-5, where the strain ε is labelled as "Factor A". This graph is used in conjunction with a stress-strain temperature graph for a specific material, as shown in Fig. A-6 and Fig. A-7, to determine the allowable external pressure for an assumed wall thickness. These graphs are applicable for both cylindrical and spherical vessels. An example in Chapter V illustrates the use of these graphs.

4.2.2 Design of Stiffening Rings

Circumferential stiffening rings are used to reduce the free length of a cylindrical shell and thus the wall thickness. In designing such stiffeners, each ring is considered to resist the external load for a distance of $L_s/2$ on either side, where L_s is the spacing between rings. Thus the load per unit length on the ring at collapse is equal to L_s x P and substituting in Eq. (4.14) taking L_s as unity, gives

$$L_s P = 2.2\ E\left(\frac{t}{D}\right)^3$$

Rewriting the above equation in terms of the moment of inertia of the ring cross-section, $I = \frac{t^3}{12}$ per unit length, results in

$$L_s P = (2.2)(12)\frac{E}{D^3}\frac{t^3}{12} = (2.2)(12)\frac{EI}{D^3}$$

$$I = \frac{L_s P D^3}{(2.2)(12)E} = \frac{L_s D^2 t}{(1.1)(12)E}\left(\frac{PD}{2t}\right)$$

and substituting $S = PD/2t$ yields

$$I = \frac{L_s D^2 t \quad S}{(1.1)(12)E} = \frac{L_s D^2 t\ \varepsilon}{(1.1)(12)} \qquad (4.20)$$

The moment of inertia of the stiffening ring and the shell plate both contribute to prevent collapse of the vessel. Timoshenko [24] has shown that the combined moment of inertia of the shell and stiffener may be considered as an equivalent thickness

$$t' = t + \frac{A_s}{L_s} \qquad (4.21)$$

where A_s is the cross-sectional area of one circumferential ring. Thus substituting Eq. (4.21) in Eq. (4.20) gives

$$I = \frac{L_s D^2 t' \varepsilon}{13.2} \tag{4.22}$$

The ASME Code formula is the same as the above except in the denominator, as follows [22]

$$I_s = D^2 L_s \left(t + \frac{A_s}{L_s}\right) A/14 \tag{4.23a}$$

$$I_s' = D^2 L_s \left(t + \frac{A_s}{L_s}\right) A/10.9 \tag{4.23b}$$

where

I_s = required moment of inertia of the stiffening ring cross-section about its neutral axis parallel to the axis of a cylindrical shell, in^4 (mm^4)

I_s' = required moment of inertia of a combined ring-shell cross-section about its neutral axis parallel to the axis of a cylindrical shell, in^4 (mm^4)

A_s = cross-sectional area of the ring, in^2 (mm^2)

A = factor determined from stress-strain diagram of a particular material (Fig. A-6 or Fig. A-7).

Stiffening rings may be placed inside or outside of the shell and can be of any structural shape, such as of a bar, angle, channel, beam, etc. Often in distillation columns, internal tray rings, besides their prime purpose to support the trays, are also considered to act like stiffeners to the shell.

4.3 COMPONENT DESIGN

In order to function properly in a plant, the design of pressure vessels incorporates a number of different standard components. The most important components are heads, nozzles, flanges, and vessel supports. Acceptable component attachments to the vessel is shown in Fig. 4.1. It is apparent that each component serves a specific purpose and its design is influenced by its functional requirements.

4.3.1 Vessel Heads

The most common types of vessel heads are of ellipsoidal, hemispherical and conical shape. The symbols, defined below, are used in the subsequent formulas as given by ref. [22], and are shown in Fig. 4.2.

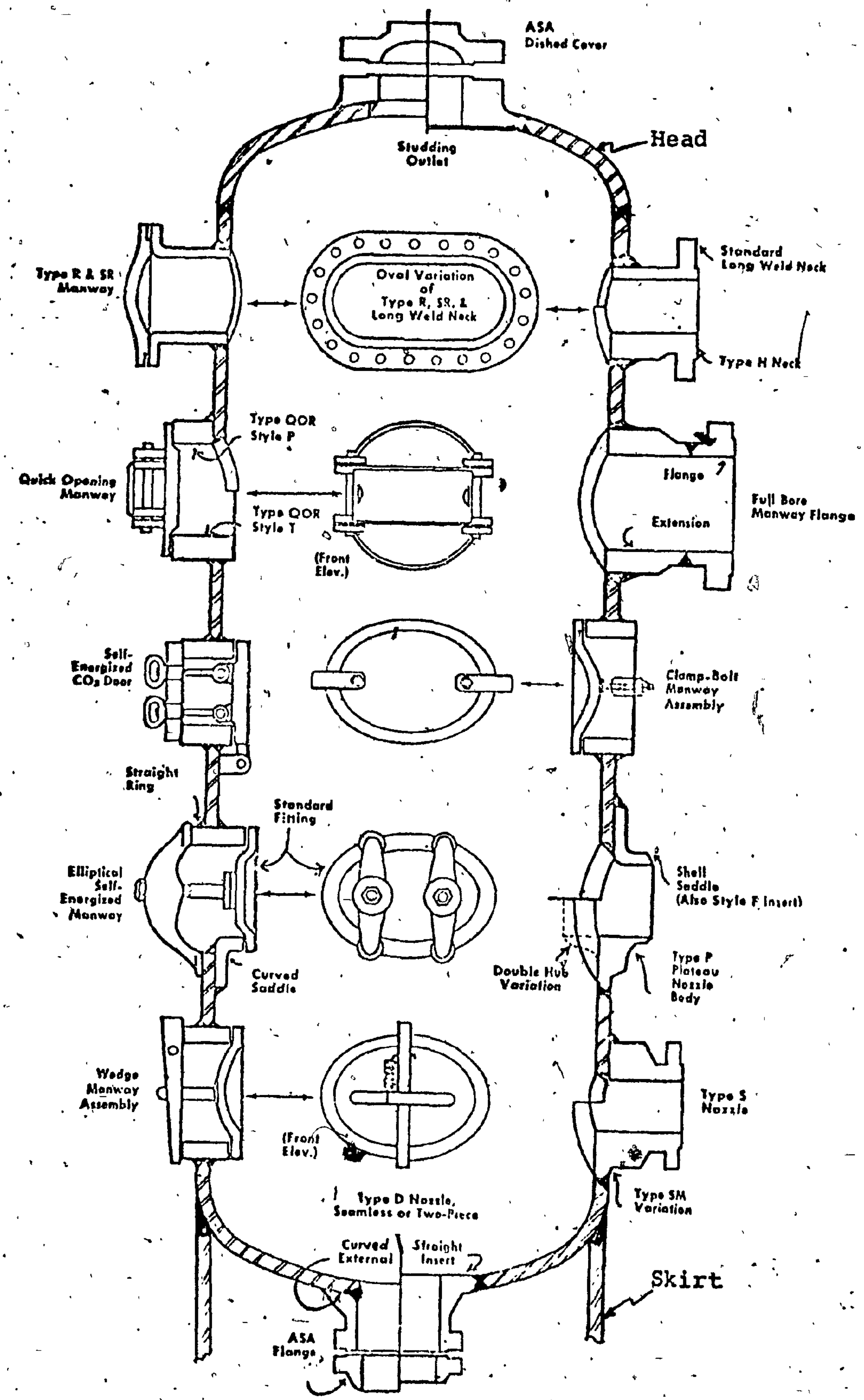


FIG. 4.1 Components of Pressure Vessels
(Adapted from the Lenape Catalogue)

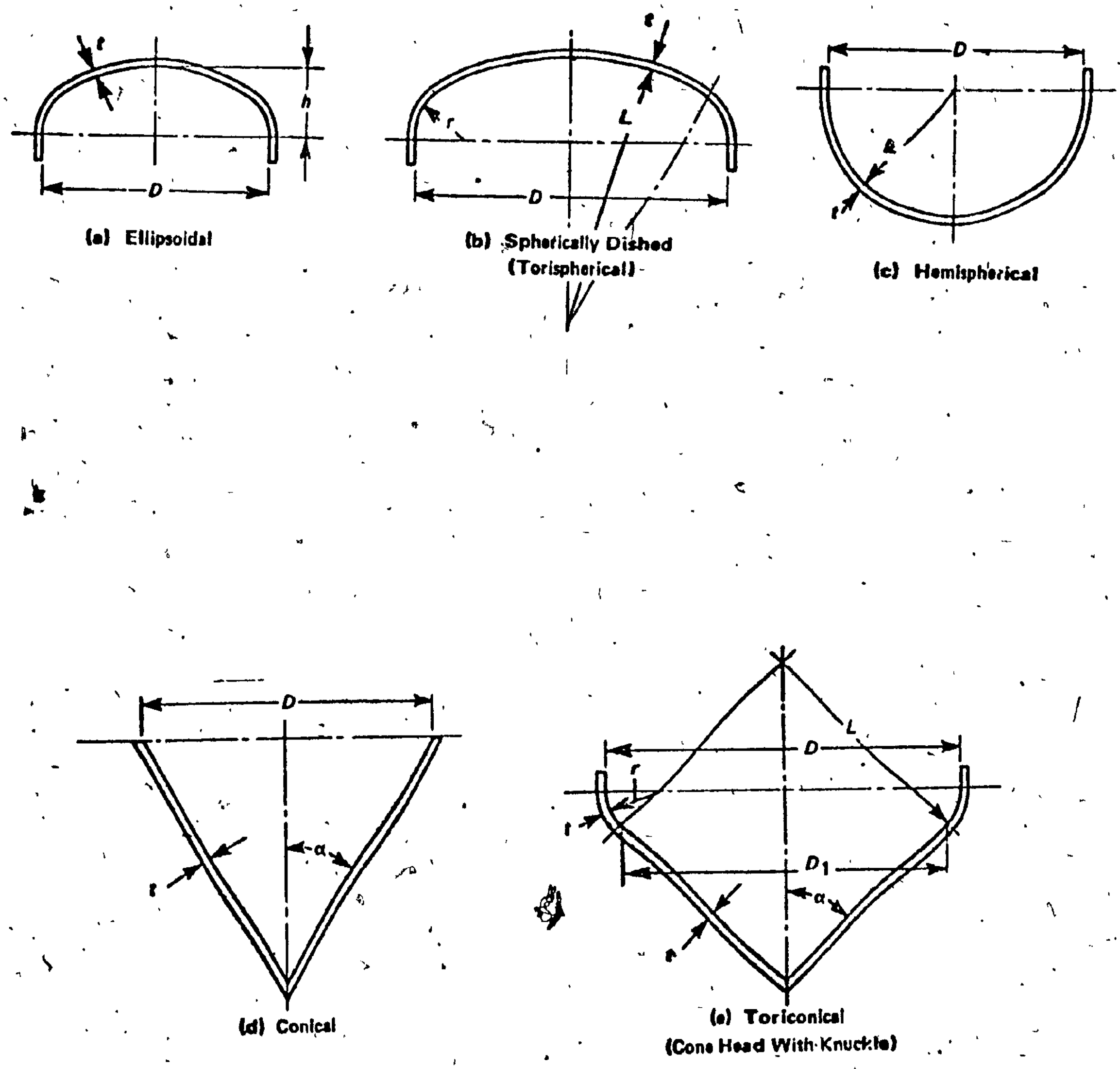


FIG. 4.2 Principal Dimensions of Typical Heads [22]

t = required thickness of head, in(mm)

P = internal design pressure, psi(kPa)

D = head diameter, in(mm)

S = maximum allowable stress, psi(MPa)

E = efficiency of weld joint (see Fig. A-2)

r = inside knuckle radius, in(mm)

L = inside spherical or crown radius for torispherical and hemispherical heads, in(mm)

h = inside depth of ellipsoidal head measured from the head-bend line (tangent line), in(mm)

α = one-half of the (apex) angle of the cone

Ellipsoidal head:

The required thickness of an ellipsoidal head is given by

$$t = \frac{PDK}{2SE - 0.2P} \qquad (4.24)$$

$$t = \frac{PDK}{(1000)2SE-0.2P} \qquad (4.24^*)$$

where

$$K = \frac{1}{6}\left[2 + \left(\frac{D}{2h}\right)^2\right]$$

W = total design bolt load for gasket seating

H_D = hydrostatic end force on area inside of flange at operating conditions

H_G = gasket load at operating conditions

H_T = hydrostatic end force on area of flange face inside of gasket at operating conditions

h_D = radial offset between force H_D and bolt center line

h_G = radial offset between force H_G and bolt center line

h_T = radial offset between force H_T and bolt center line

For gasket seating conditions, $W = \frac{1}{2}(A_m + A_b)S_a$ (Fig.A-3) and the total flange moment is

$$M = Wh_G \tag{4.28}$$

and under operating conditions, the total flange moment is

$$M_o = H_D h_D + H_T h_T + H_G h_G \tag{4.29}$$

The stresses considered are defined as

S_H = longitudinal bending stress in hub adjacent to flange.

S_R = radial stress in flange

S_T = tangential stress in flange

These stresses are limited in relation to the design stress S_{fo} for the flange material:

$$S_H \leq 1.5\ S_{fo}$$
$$S_R \leq S_{fo}$$
$$S_T \leq S_{fo}$$

$$(S_H+S_R)/2 \leq S_{fo}$$
$$(S_H+S_T)/2 \leq S_{fo}$$

To facilitate the design of custom flanges, the Taylor Forge method has been published in a convenient computation sheet, as shown in Fig. A-3.

4.3.4 Vessel Support

Pressure vessels may be horizontal or vertical depending on the intended application and space restrictions. Horizontal vessels are usually supported by means of saddles which are designed to alleviate severe stress concentration. The procedure for supporting horizontal vessels on two saddles is discussed in ref. [26].

Vertical vessels can be supported in several ways such as by skirts, legs or attached lugs. Tall vessels are usually supported on skirts while relatively smaller vessels on legs or lugs. The Skirt type support is the only one capable of resisting large bending moments generated by wind or seismic loads.

Also, the local stresses which are developed around the interface of the skirt and vessel are evenly distributed.

The skirt wall should be thick enough to withstand the weight of the vessel and a maximum possible external load (wind gusts, strong earthquakes). Thus, an adequate skirt thickness can be obtained by the following formula [27].

$$t = \frac{12\ M_T}{R^2 \pi SE} + \frac{W_V}{D\pi SE} \qquad (4.30)$$

$$t = \frac{M_T}{R^2 \pi SE} + \frac{W_V}{D\pi SE} \qquad (4.30^*)$$

where

t = required thickness of skirt, in (mm)

M_T = total moment at the skirt-to-head joint, ft-lb (N-mm)

R = outside radius of skirt, in (mm)

S = stress value of the head or skirt material, whichever is smaller, psi (MPa)

E = efficiency of skirt-to-head joint (Fig. A-2)

W_V = weight of the vessel in operating conditions, lb_f (N)

In addition to the thickness, the elastic stability of the skirt must also be checked to ensure that buckling does not occur. The allowable critical compressive stress at which skirt buckling does not occur under axial compression, is given by

$$S_c = 1.5 \times 10^6 \left(\frac{t}{R}\right) \leq \frac{1}{3} \text{ yield point} \qquad (4.31)$$

Eq. (4.31) shows that the compressive stresses S_c developed on a skirt, with radius R and thickness t, should not exceed one-third the yield point value of the skirt material. This empirical formula is not applicable to vessels constructed to ASME Code requirements. Design rules and general recommendations representing the current state of the art may be found in [22]. Further discussion on vertically supported vessels, including the base of the skirt, can also be found in [28].

CHAPTER V

ILLUSTRATIVE EXAMPLE

CHAPTER V

ILLUSTRATIVE EXAMPLE

An effluent water tower, shown in Fig. 5.1, is required to operate at a maximum internal pressure of 95 psig and temperature of 340^{o}F. The tower also will be subjected to full vacuum periodically. A corrosion allowance of 1/4 in. and carbon steel material are specified. Vessel design to comply with Section VIII, Div. 1, of the ASME Code.

ASTM A-516 Gr. 70 is the selected material for shell and heads, and nozzle necks.

Since all welded joints are fully radiographed for Type 1, an efficiency of 100% is taken from Fig. A-2.

5.1 SUMMARY OF DESIGN DATA

Design pressure	:	95 psig + full vacuum
Design temperature	:	340^{o}F
Material	:	ASTM A-516 GR.70
Corrosion allowance	:	1/4 in.
Bottom part diameter	:	156 in
Top part diameter	:	84 in
Weld joint efficiency	:	100%

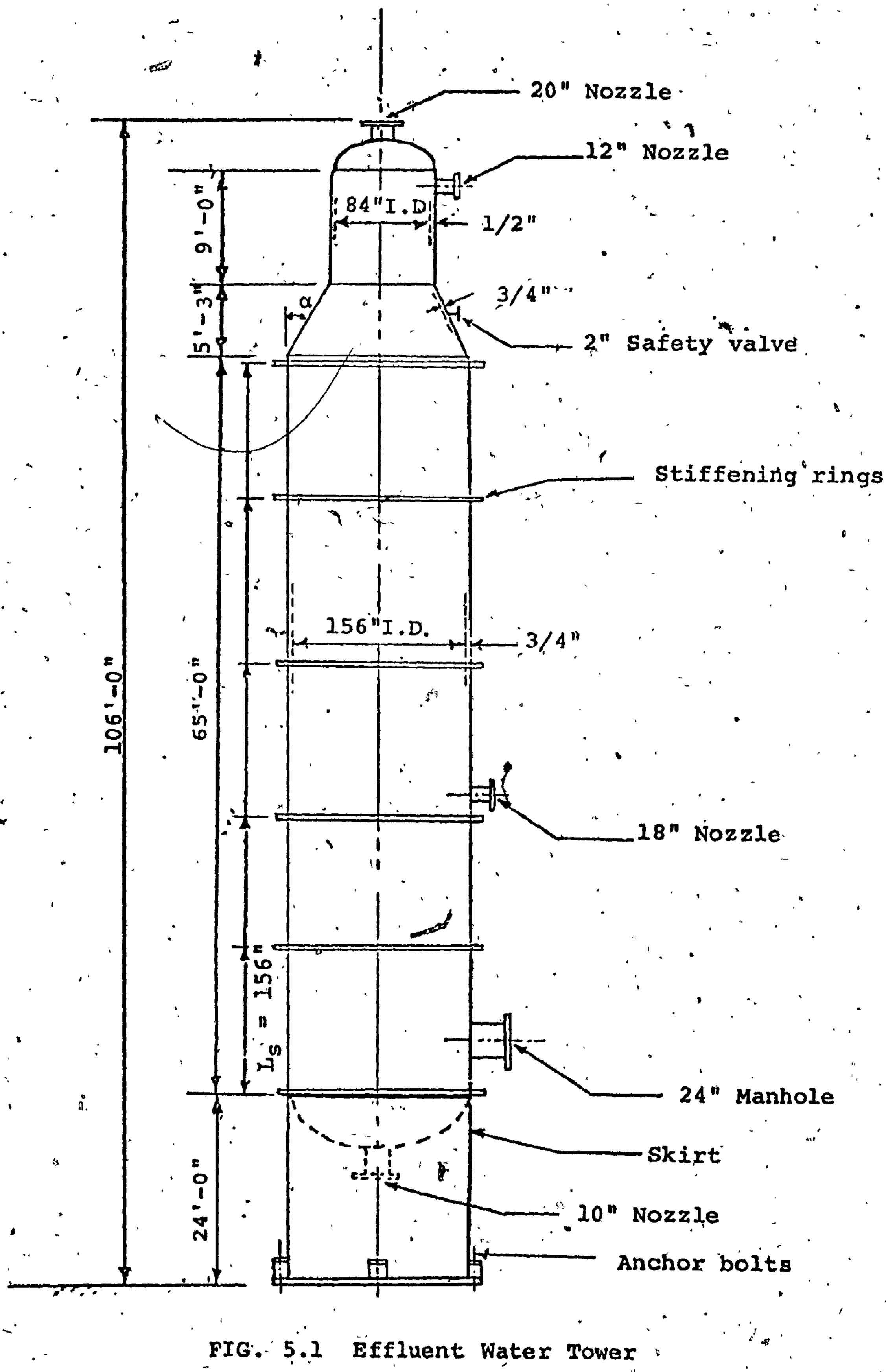


FIG. 5.1 Effluent Water Tower

5.2 REQUIRED THICKNESSES UNDER INTERNAL PRESSURE

Bottom Head (2:1 ratio)

$$t = \frac{PD}{2SE - 0.2P}$$

where

P = 95 psig (including hydrostatic head of operating fluid)

D = 156 in.

E = 1.0

S = 17,500 psi, (from Fig. A-8 at temperature range -20 to 650°F)

Therefore

$$t = \frac{(95)(156.50)}{(2)(17,500)(1) - 0.2(95)} = 0.425 \text{ in.}$$

To this thickness, the specified corrosion allowance is added, i.e.

$$t = 0.425 + 0.250 = 0.675 \text{ in.}$$

Use head of 3/4 in. minimum thickness after forming.

Shell of 156 in. diameter:

Similarly

$$t = \frac{PR}{SE - 0.6P} = \frac{(95)(78.25)}{(17,500)(1)-0.6)(95)} =$$

$$= 0.426 \text{ in.}$$

and $$t = 0.426 + 0.250 = 0.676 \text{ in.}$$

Use 3/4 in. plate.

Shell of 84 in. diameter:

$$t = \frac{PR}{SE - 0.6P} = \frac{(95)(42.25)}{(17,500)(1)-(0.6)(95)} =$$

$$= 0.230 \text{ in.}$$

and $$t = 0.230 + 0.250 = 0.480 \text{ in.}$$

Use 1/2 in. plate.

Cone:

$$\tan\alpha = \frac{78 - 42}{(5.2)(12)} = 0.576$$

$$\alpha = \tan^{-1}(0.576) = 30^{\circ}$$

$$t_r = \frac{PD}{2\cos\alpha(SE - 0.6P)} =$$

$$= \frac{(95)(156.50)}{(2)(0.867)[(17,500)(1)-(0.6)(95)]} =$$

$$= 0.492 \text{ in.}$$

and $$t = 0.492 + 0.250 = 0.762 \text{ in.}$$

Use 3/4 in. plate.

<u>Top Head:</u> (2:1 ratio)

$$t = \frac{PD}{2SE - 0.2P} = \frac{(95)(84.25)}{(2)(17,500)(1)-(0.2)(95)} =$$

$$= 0.228 \text{ in.}$$

$$t = 0.228 + 0.25 = 0.478 \text{ in.}$$

Use head of 1/2 in. minimum thickness after forming.

5.3 DESIGN AT VACUUM CONDITIONS

Check large diameter shell for external pressure of 14.7 psi (full vacuum) with t = 1/2 in, L = 65 ft and D_o = 157.5 in.

$$\frac{L}{D_o} = \frac{(65)(12)}{157.5} = 4.952$$

$$\frac{D_o}{t} = \frac{157.5}{0.50} = 315$$

From Fig. A-5, Factor A = 0.000046, and from Fig. A-7 the value of A falls to the left end of the chart and according to ref. [22], the allowable external pressure is given by

$$P_a = \frac{2AE}{3(D_o/t)} = \frac{(2)(46)(10^{-6})(29)(10^{6})}{3(315)} =$$

$$= 2.8 \text{ psi}$$

Thus, it is apparent that the shell does not comply with requirements of externally applied pressure. Therefore, the shell must be stiffened with circumferential rings.

Try with 6 rings spaced at L_s = 156 in. apart.

$$\frac{L_s}{D_o} = \frac{156}{157.5} = 0.99$$

$$\frac{D_o}{t} = \frac{157.5}{0.50} = 416$$

From Fig. A-5, Factor A = 0.00024 and from Fig. A-7 and at temperature of 340°F, Factor B = 3,500, and from ref. [22], the allowable external pressure is given by

$$P_a = \frac{4B}{3(D_o/t)} = \frac{(4)(3,500)}{3(315)} = 14.8 \text{ psi} > 14.7$$

Stiffening Rings:

The required moment of inertia of a circumferential ring must be at least equal to

$$I_s = [D_o^2 L_s (t + A_s/L_s) A]/14$$

Try a $\boxed{}$ 1 x 8 in. ring of the same material with shell. (A_s = 1 x 8 = 8 in^2).

In order to find the value of A, the value of B must be calculated [22], i.e.,

$$B = \frac{3}{4}\left[\frac{PD_o}{t + A_s/L_s}\right] = \frac{3}{4}\left[\frac{(14.7)(157.5)}{0.50 + 8/156}\right] = 3,150$$

From Fig. A-7, Factor A can be found, i.e.,

$$A = 0.000225$$

Therefore

$$I_s = [(157.5^2)(156)(0.50+8/156)(0.000225)]/14 =$$

$$= \frac{469}{14} = 33.52 \text{ in.}^4$$

The moment of inertia of the ring cross-section is

$$I = \frac{8^3 \times 1}{12} = 42.6 \text{ in.}^4 > 33.52$$

Hence, 6 stiffening rings of 1 x 8 in. spaced at 156 in, would be satisfactory for the large diameter shell to withstand full vacuum conditions. For the small diameter shell, calculations show that there is no need of stiffeners. Stiffening rings could also be of any standard structural shape such as, beams, channels, angles, etc.

5.4 NOZZLE REINFORCEMENT

24" Manhole

The manhole neck is rolled from 1 in. plate of the same material as the shell. The required thickness of the neck is

$$t_{rn} = \frac{PR}{S - 0.6P} = \frac{(95)(12.250)}{17,500 - (0.6)(95)} = 0.067 \text{ in.}$$

From Fig. 5.2, the area of reinforcement required at corroded conditions is:

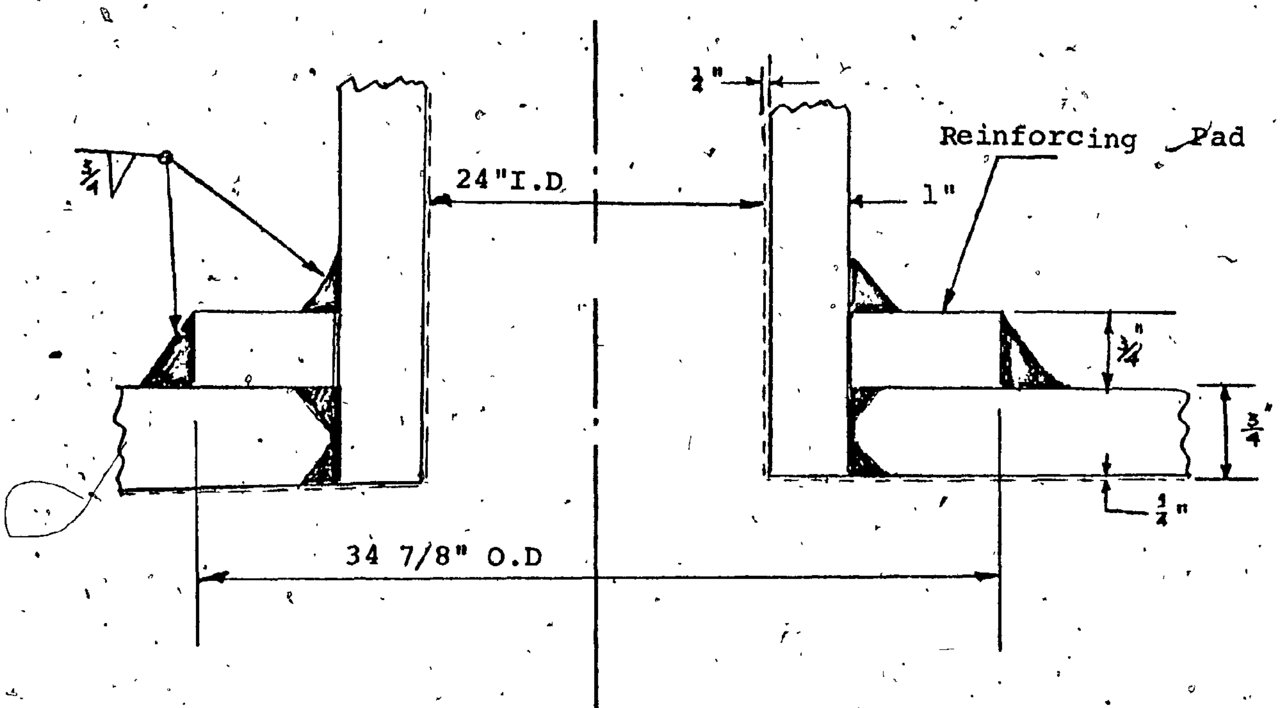


FIG. 5.2 Example of Nozzle Reinforcement

$$A = d \times t_r = 24.5 \times 0.426 = 10.437 \text{ in}^2$$

Area of reinforcement provided is:

$$A_1 = [(t-c) - t_r]d = [(0.75-.25) - 0.426]\ 24.5 =$$

$$= 1.813 \text{ in}^2$$

$$A_2 = [(t_n-c) - t_{rn}]5t = [(1-0.25) - 0.067](5)(0.5) =$$

$$= 1.708 \text{ in}^2$$

$$A_4 = (2)(.75^2) = 1.125 \text{ in}^2$$

Therefore, the total area provided is 4.646 in.^{2}

Area that has to be supplied by reinforcing pad is

$$A_5 = 10.437 - 4.646 = 5.791 \text{ in}^2.$$

A pad of 0.75 in. thick and 34.875 in. O.D. is used.

$$A_5 = (34.875-26)(0.75) = 6.656 \text{ in}^2$$

Thus,

$$A_1 + A_2 + A_4 + A_5 = 4.646 + 6.656 = 11.302 \text{ in}^2 > 10.437$$

Nozzle reinforcement must also be checked at test conditions, i.e., without considering corrosion allowance. Similarly, other nozzles are reinforced.

APPENDIX

FIGURES

Guide to ASME Sec. VIII, Div. 1

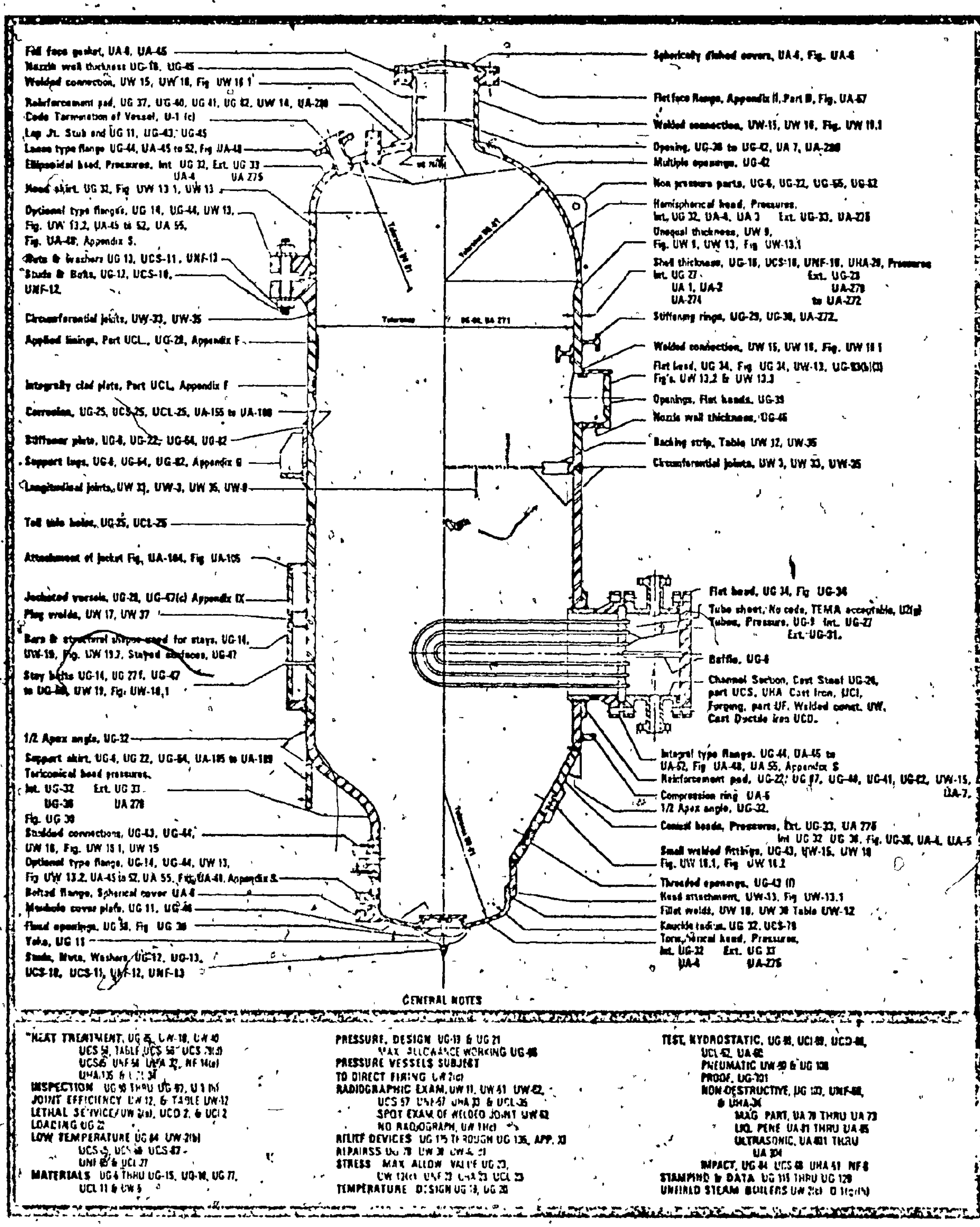


FIG. A-1 Guide to ASME Code, Section VIII, Div. 1 [29]

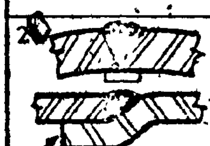
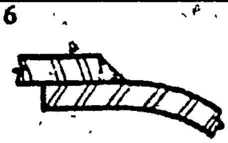
WELDED JOINTS					
TYPES			JOINT EFFICIENCY, E a. When the Joint:		
			Fully Radio-graphed	b. Spot Examined	c. Not Examined
1		Butt joints as attained by double-welding or by other means which will obtain the same quality of deposited weld metal on the inside and outside weld surface. Backing strip if used shall be removed after completion of weld.	1.00	0.85	0.70
2	 For circumferential joint only	Single-welded butt joint with backing strip which remains in place after welding	0.90	0.80	0.65
3		Single-welded butt joint without use of backing strip	–	–	0.60
4		Double-full fillet lap joint	–	–	0.55
5		Single-full fillet lap joint with plug welds	–	–	0.50
6		Single full fillet lap joint without plug welds	–	–	0.45

FIG. A-2 Efficiency of Welded Joints [27]

WELDING NECK FLANGE DESIGN

DESIGN CONDITIONS		GASKET and BOLTING CALCULATIONS		(From Design Tables 2 and 3)
Design Pressure, P		Gasket Details	Facing Details	$N =$
Design Temperature				$b =$
Flange Material				$y =$
Bolting Material				$m =$
Corrosion Allowance		$W_{m2} = b\pi Gy$ =	$A_m =$ greater of W_{m2}/S_a or W_{m1}/S_b =	
Allowable Stresses — Flange: Design Temp., S_{fo}		$H_p = 2b\pi GmP$ =	A_b =	
Allowable Stresses — Flange: Atm. Temp., S_{fa}		$H = G^2\pi P/4$ =	$W = .5(A_m + A_b)S_a$ =	
Allowable Stresses — Bolting: Design Temp., S_b		$W_{m1} = H_p + H$ =	W_{m1} =	
Allowable Stresses — Bolting: Atm. Temp., S_a		Gasket Width Check (Raised Face ONLY): $N_{min} = A_bS_a/2y\pi G$ =		

CONDITION	LOAD	X LEVER ARM	= MOMENT
Operating	$H_D = \pi B^2P/4$ =	$h_D = R + .5g_1$ =	$M_D = H_Dh_D$ =
	$H_G =$ H_p =	$h_G = .5(C - G)$ =	$M_G = H_Gh_G$ =
	$H_T = H - H_D$ =	$h_T = .5(R + g_1 + h_G)$ =	$M_T = H_Th_T$ =
			M_o =
Gasket Seating	$H_G = W$ =	$h_G = .5(C - G)$ =	M_o =

Allowable Stress	STRESS CALCULATION—*Operating* Conditions (use M)			
$1.5\ S_{fo}$	Long. Hub, $S_H = fM/\lambda g_1^2$			
S_{fo}	Radial Flg., $S_R = \beta M/\lambda t^2$			
S_{fo}	Tang. Flg., $S_T = (MY/t^2) - ZS_R$			
S_{fo}	greater of $.5(S_H + S_R)$ or $.5(S_H + S_T)$			

Allowable Stress	STRESS CALCULATION—Gasket Seating (use M)			
$1.5\ S_{fa}$	Long. Hub, $S_H = fM/\lambda g_1^2$			
S_{fa}	Radial Flg., $S_R = \beta M/\lambda t^2$			
S_{fa}	Tang. Flg., $S_T = (MY/t^2) - ZS_R$			
S_{fa}	greater of $.5(S_H + S_R)$ or $.5(S_H + S_T)$			

SHAPE CONSTANTS (From Design Table 4 and Design Charts 1, 2 and 5)			
$K = A/B$ =		h/h_o =	
T =		F =	
Z =		V =	
Y =		f =	
U =		$e = F/h_o$ =	
g_1/g_o =		$d = \frac{U}{V}h_og_o^2$ =	
$h_o = \sqrt{Bg_o}$ =			

OTHER STRESS FORMULA FACTORS			
t (assumed)			
$\alpha = te + 1$			
$\beta = (4/3)(te) + 1$			
$\gamma = \alpha/T$			
$\delta = t^3/d$			
$\lambda = \gamma + \delta$			
$M = M_o/B$			
$M = M_o/B$			

If bolt spacing exceeds $2a + t$, multiply M_o and M_o in above equations by: $\sqrt{\frac{\text{Bolt spacing}}{2a + t}}$

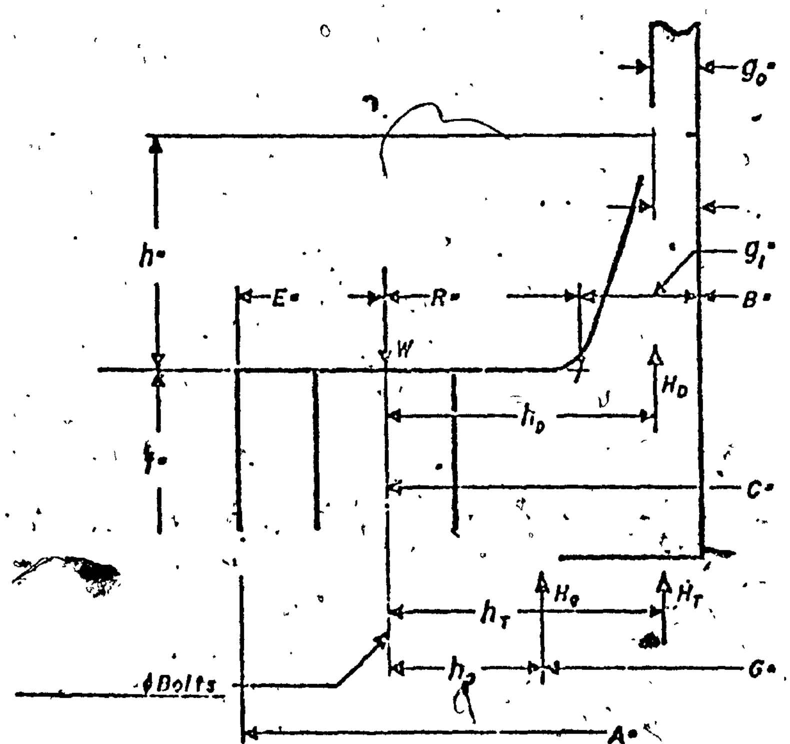


FIG. A-3 Computation Sheet for Flange Design [25]

Poor copy

1. Applied Loads*

Radial load, P = ——— lb.
Circ. Moment, Mc = ——— in. lb.
Long. Moments, ML = ——— in. lb.
Torsion Moment, MT = ——— in. lb.
Shear Load, Vc = ——— lb.
Shear Load, VL = ——— lb.

2. Geometry

Vessel thickness, T = ——— in.
Attachment radius, r_o = ——— in.
Vessel radius, R_m = ——— in.

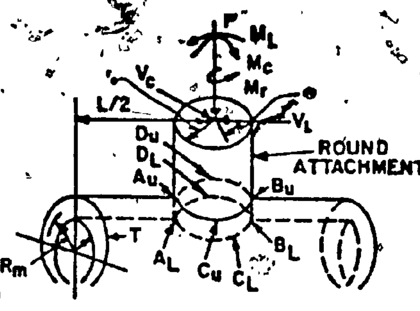
3. Geometric Parameters

$\gamma = \frac{R_m}{T} =$ ———

$\beta = (0.875)\frac{r_o}{R_m} =$ ———

Stress Concentration due to:
a) membrane load, Kn ———
b) bending load, Kb ———

*NOTE: Enter all force values in accordance with sign convention



From Fig.	Read curves for	Compute absolute values of stress and enter result:	STRESSES – if load is opposite that shown, reverse signs shown							
			Au	AL	Bu	BL	Cu	CL	Du	DL
3C	$\frac{N_\phi}{P/R_m}$ =	$K_n\left(\frac{N_\phi}{P/R_m}\right)\cdot\frac{P}{R_mT}$ =	−	−	−	−	−	−	−	−
1C	$\frac{M_\phi}{P}$ =	$K_b\left(\frac{M_\phi}{P}\right)\cdot\frac{6P}{T^2}$ =	−	+	−	+	−	+	−	+
3A	$\frac{N_\phi}{M_c/R_m^2\beta}$ =	$K_n\left(\frac{N_\phi}{M_c/R_m^2\beta}\right)\cdot\frac{M_c}{R_m^2\beta T}$ =					−	−	+	+
1A	$\frac{M_\phi}{M_c/R_m\beta}$ =	$K_b\left(\frac{M_\phi}{M_c/R_m\beta}\right)\cdot\frac{6M_c}{R_m\beta T^2}$ =					−	+	+	−
3B	$\frac{N_\phi}{M_L/R_m^2\beta}$ =	$K_n\left(\frac{N_\phi}{M_L/R_m^2\beta}\right)\cdot\frac{M_L}{R_m^2\beta T}$ =	−	−	+	+				
1B or 1B-1	$\frac{M_\phi}{M_L/R_m\beta}$ =	$K_b\left(\frac{M_\phi}{M_L/R_m\beta}\right)\cdot\frac{6M_L}{R_m\beta T^2}$ =	−	+	+	−				
Add algebraically for summation of ϕ stresses, σ_ϕ =										
4C	$\frac{N_x}{P/R_m}$ =	$K_n\left(\frac{N_x}{P/R_m}\right)\cdot\frac{P}{R_mT}$ =	−	−	−	−	−	−	−	−
2C	$\frac{M_x}{P}$ =	$K_b\left(\frac{M_x}{P}\right)\cdot\frac{6P}{T^2}$ =	−	+	−	+	−	+	−	+
4A	$\frac{N_x}{M_c/R_m^2\beta}$ =	$K_n\left(\frac{N_x}{M_c/R_m^2\beta}\right)\cdot\frac{M_c}{R_m^2\beta T}$ =					−	−	+	+
2A	$\frac{M_x}{M_c/R_m\beta}$ =	$K_b\left(\frac{M_x}{M_c/R_m\beta}\right)\cdot\frac{6M_c}{R_m\beta T^2}$ =					−	+	+	−
4B	$\frac{N_x}{M_L/R_m^2\beta}$ =	$K_n\left(\frac{N_x}{M_L/R_m^2\beta}\right)\cdot\frac{M_L}{R_m^2\beta T}$ =	−	−	+	+				
2B or 2B-1	$\frac{M_x}{M_L/R_m\beta}$ =	$K_b\left(\frac{N_x}{M_L/R_m\beta}\right)\cdot\frac{6M_L}{R_m\beta T^2}$ =	−	+	+	−				
Add algebraically for summation of X stresses, σ_x =										
Shear stress due to Torsion, M_T		$\tau_{\phi x} = \tau_{x\phi} = \frac{M_T}{2\pi r_o^2 T}$	+	+	+	+	+	+	+	+
Shear stress due to load, V_c		$\tau_{x\phi} = \frac{V_c}{\pi r_o T}$ =	+	+	−	−				
Shear stress due to load, V_L		$\tau_{x\phi} = \frac{V_L}{\pi r_o T}$ =					−	−	+	+
Add Algebraically for summation of shear stresses, τ =										
COMBINED STRESS INTENSITY, S										
1) When σ_ϕ & σ_x have like signs		$S = \frac{1}{2}\left[\sigma_\phi + \sigma_x \pm \sqrt{(\sigma_\phi - \sigma_x)^2 + 4\tau^2}\right]$								
2) When $\tau = 0$		S = largest of σ_ϕ, σ_x or $\lvert\sigma_\phi - \sigma_x\rvert$								
3) When σ_ϕ & σ_x have unlike signs		$S = \sqrt{(\sigma_\phi - \sigma_x)^2 + 4\tau^2}$								

FIG. A-4 Computation Sheet for Local Stress in Cylindrical Shells [20]

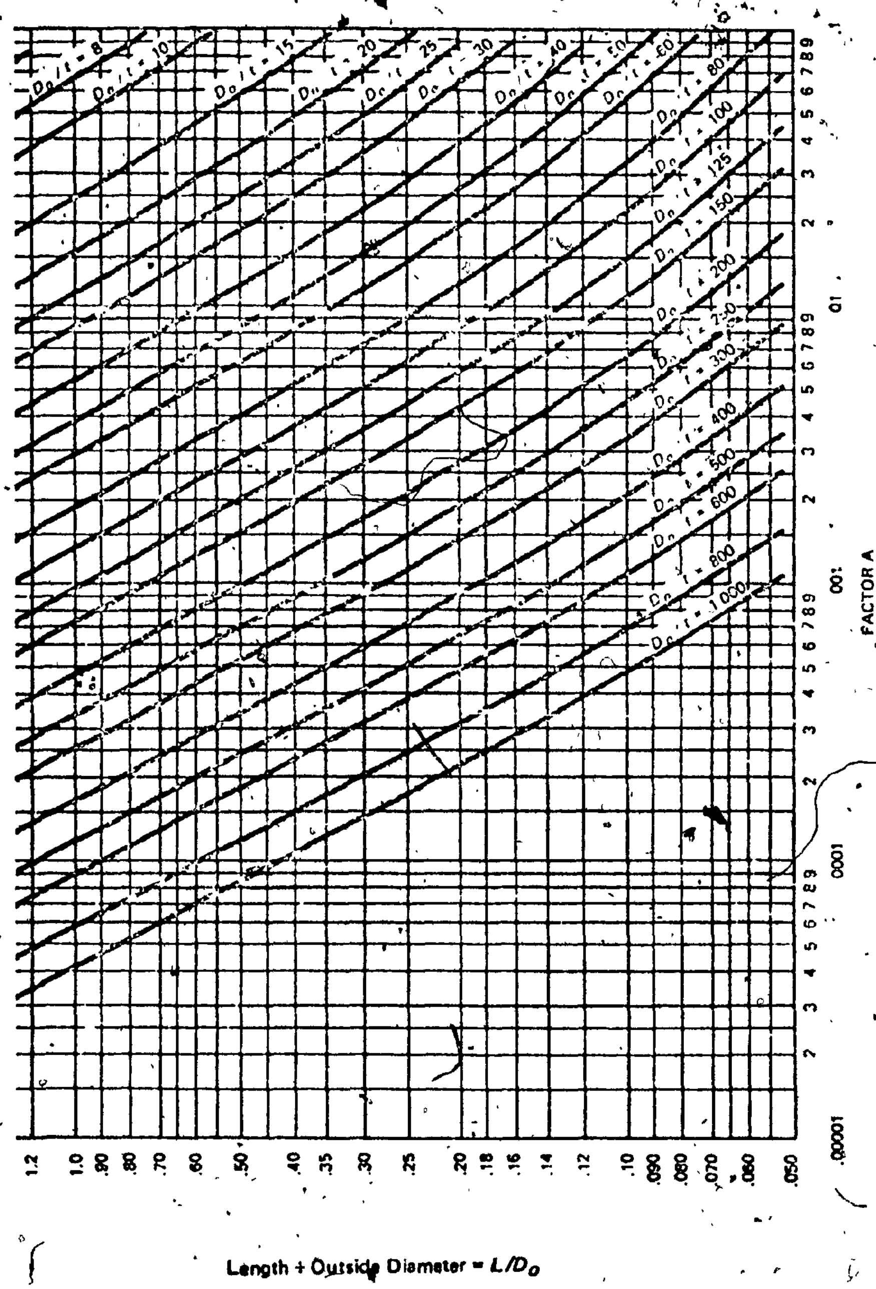


FIG. A-5 (1) Geometric Chart for Cylindrical Vessels Under External Pressure [22]

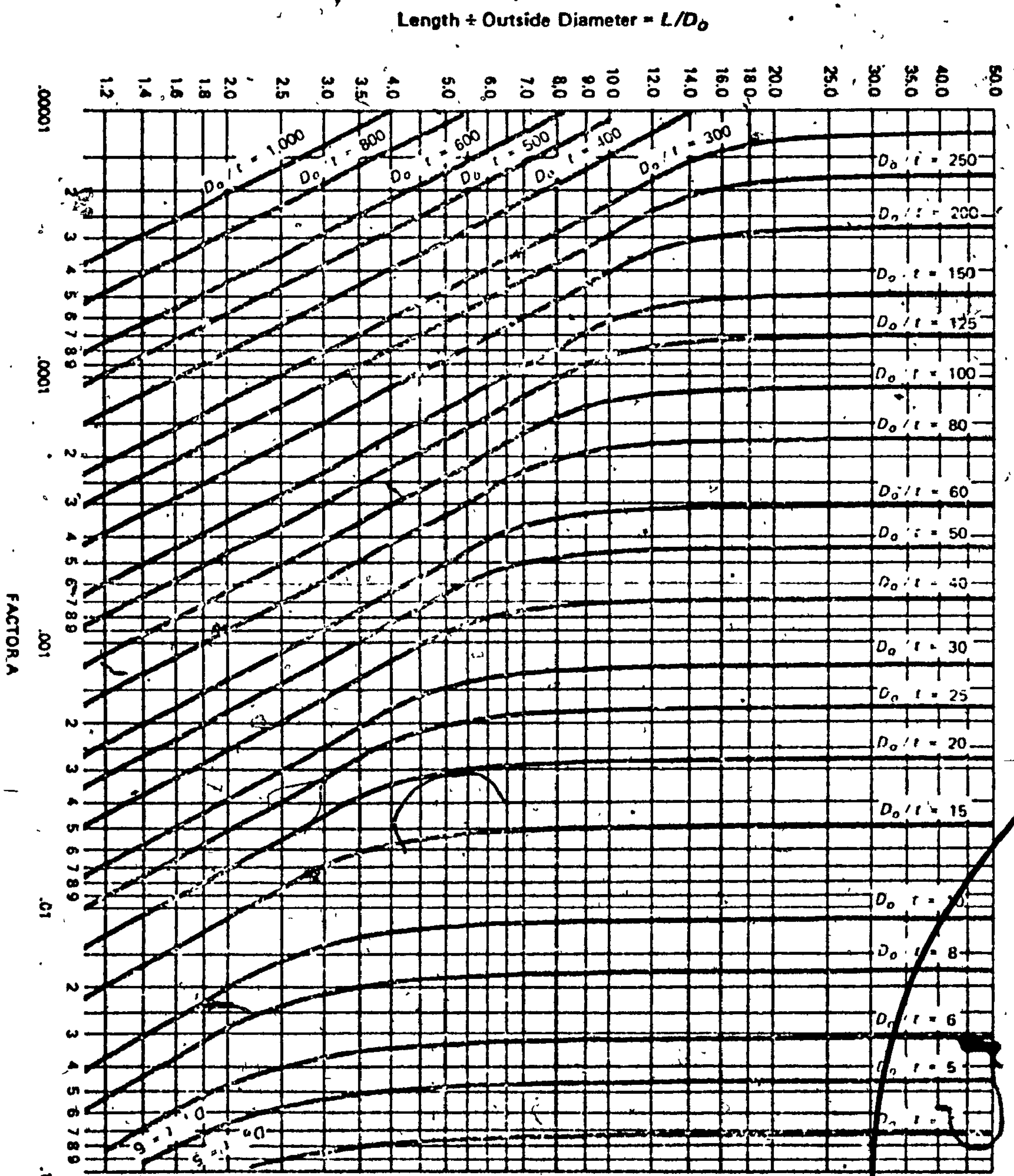


FIG. A-5 (2) Geometric Chart for Cylindrical Vessels Under External Pressure [22]

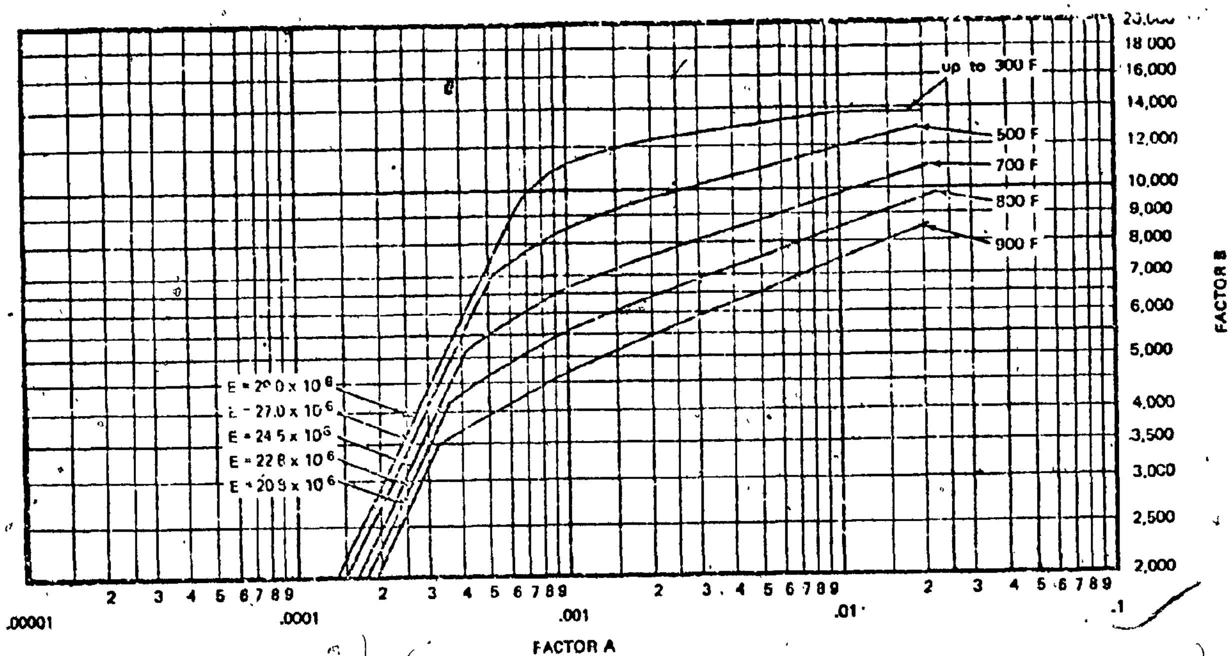


FIG. A-6 Chart for Determining Shell Thickness of Cylindrical and Spherical Vessels Under External Pressure When Constructed of Carbon or Low-Alloy Steels With a Specified Yield Strength of 24,000-29,000 psi [22]

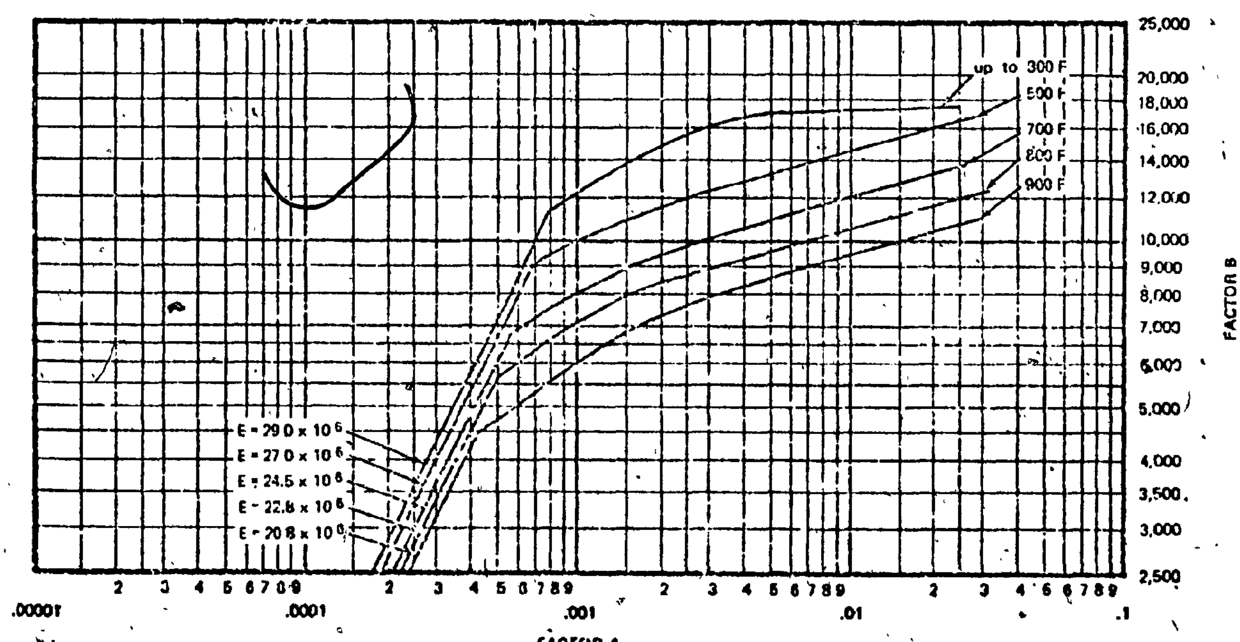


FIG. A-7 Chart for Determining Shell Thickness of Cylindrical and Spherical Vessels Under External Pressures When Constructed of Carbon or Low-Alloy Steels With a Specified Yield Strength of 30,000 - 38,000 psi [22]

Spec. No.	Grade	Nominal Composition	P-No.	Group No.	Specified Minimum Yield	Specified Minimum Tensile
Bolting—All Carbon & Low Alloy Steels						
SA-193	B5	5 Cr-½ Mo			80.0	100.0
	B7	1 Cr-⅕ Mo ≤2½ in.	...	...	105.0	125.0
		1 Cr-⅕ Mo <2½ in. and ≤4 in.	...	...	95.0	115.0
		1 Cr-⅕ Mo >4 in.	...	...	75.0	100.0
	B7M	1 Cr-⅕ Mo ≤2½ in.	...	...	80.0	100.0
	B16	1 Cr-½ Mo-V ≤2½ in.	...	...	105.0	125.0
		1 Cr-½ Mo-V >2½ in. and ≤4 in.	...	...	95.0	110.0
		1 Cr-½ Mo-V >4 in.	...	...	85.0	100.0
SA-307	B	C	...	...	...	55.0
SA-320	L7, L43	Cr-Mo and Ni-Cr-Mo			105.0	125.0
SA-325	1	C	...	...	77.0	105.0
SA-354	BB	...	...	...	78.0	100.0
	BC	...	...	...	99.0	115.0
	BD	...	...	...	125.0	150.0
SA-449	1 in. & under	C	...	...	92.0	120.0
	>1 in. and <1½ in.	C	...	...	81.0	105.0
	>1½ in. and <3 in.	C	...	...	58.0	90.0
Carbon Steel Plates and Sheets						
SA-36	...	C-Mn-Si	1	1	36.0	58.0
SA-283	A	C	1	1	24.0	45.0
	B	C	1	1	27.0	50.0
	C	C	1	1	30.0	55.0
	D	C	1	1	33.0	60.0
SA-285	A	C	1	1	24.0	45.0
	B	C	1	1	27.0	50.0
	C	C	1	1	30.0	55.0
SA-299	...	C-Mn-Si	1	2	40.0/42.0	75.0
SA-414	A	C	1	1	24.0	45.0
	B	C	1	1	27.0	50.0
	C	C	1	1	30.0	55.0
	D	C-Mn	1	1	32.0	60.0
	E	C-Mn	1	1	35.0	65.0
	F	C-Mn	1	1	38.0	70.0
	G	C-Mn	1	2	42.0	75.0
SA-442	55	C-Mn-Si	1	1	30.0	55.0
	60	C-Mn-Si	1	1	32.0	60.0
SA-455	Up to 0.375 in.	C-Mn	1	2	37[illegible]	75.0
	0.375 in. to 0.580 in.	C-Mn	1	2	36.5	73.0
	0.58 in. to 0.750 in.	C-Mn	1	2	35.0	70.0
SA-515	55	C-Si	1	1	30.0	55.0
	60	C-Si	1	1	32.0	60.0
	65	C-Si	1	1	35.0	65.0
	70	C-Si	1	1	38.0	70.0
SA-516	55	C-Si	1	1	30.0	55.0
	60	C-Si	1	1	32.0	60.0
	65	C-Mn-Si	1	1	35.0	65.0
	70	C-Mn-Si	1	1	38.0	70.0

FIG. A-8 (1) Maximum Allowable Stress Values for Carbon and Low Alloy Steel [22]

For Metal Temperatures Not Exceeding Deg. F												
-20 to 650	700	750	800	850	900	950	1000	1050	1100	1150	1200	Specification Number
												Bolting—All Carbon & Low Alloy Steels
20.0	20.0	20.0	18.5	14.5	10.4	7.6	5.6	4.2	3.1	2.0	1.3	SA-193
25.0	25.0	23.5	21.0	17.0	12.5	8.5	4.5	...	...	...	...	SA-193
23.0	23.0	22.2	20.0	16.3	12.5	8.5	4.5	...	...	...	...	SA-193
18.8	18.8	18.8	18.0	16.3	12.5	8.5	4.5	...	...	...	...	SA-193
20.0	20.0	20.0	18.5	16.2	12.5	8.5	4.5	...	...	...	...	SA-193
25.0	25.0	25.0	25.0	23.5	20.5	16.0	11.0	6.3	2.8	...	...	SA-193
22.0	22.0	22.0	22.0	21.0	18.5	15.3	11.0	6.3	2.8	...	...	SA-193
20.0	20.0	20.0	20.0	18.8	16.7	14.3	11.0	6.3	2.8	...	...	SA-193
...	...	...	...	...	...	...	...	...	...	...	...	SA-307
...	...	...	...	...	...	...	...	...	...	...	...	SA-320
19.3	...	...	...	...	...	...	...	...	...	...	...	SA-325
19.5	...	...	...	...	...	...	...	...	...	...	...	SA-354
23.0	...	...	...	...	...	...	...	...	...	...	...	SA-354
30.0	...	...	...	...	...	...	...	...	...	...	...	SA-354
23.0	...	...	...	...	...	...	...	...	...	...	...	SA-449
20.2	...	...	...	...	...	...	...	...	...	...	...	SA-449
14.5	...	...	...	...	...	...	...	...	...	...	...	SA-449
												Carbon Steel Plates and Sheets
12.7	...	...	...	...	...	...	...	...	...	...	...	SA-36
10.4	...	...	...	...	...	...	...	...	...	...	...	SA-283
11.5	...	...	...	...	...	...	...	...	...	...	...	SA-283
12.7	...	...	...	...	...	...	...	...	...	...	...	SA-283
12.7	...	...	...	...	...	...	...	...	...	...	...	SA-283
11.3	11.0	10.3	9.0	7.8	6.5	...	...	...	...	...	...	SA-285
12.5	12.1	11.2	9.6	8.1	6.5	...	...	...	...	...	...	SA-285
13.8	13.3	12.1	10.2	8.4	6.5	...	...	...	...	...	...	SA-285
18.8	17.7	15.7	12.6	9.5	6.5	4.5	2.5	...	...	...	...	SA-299
11.3	11.0	10.3	9.0	7.8	6.5	...	...	...	...	...	...	SA-414
12.5	12.1	11.2	9.6	8.1	6.5	...	...	...	...	...	...	SA-414
13.8	13.3	12.1	10.2	8.4	6.5	...	...	...	...	...	...	SA-414
15.0	14.3	12.9	10.8	8.6	6.5	...	...	...	...	...	...	SA-414
16.2	15.5	13.8	11.4	8.9	6.5	...	...	...	...	...	...	SA-414
17.5	16.6	14.7	12.0	9.2	6.5	...	...	...	...	...	...	SA-414
18.8	17.7	15.7	12.6	9.6	6.5	...	...	...	...	...	...	SA-414
13.8	13.3	12.1	10.2	8.4	...	...	...	...	...	...	...	SA-442
15.0	14.4	13.0	10.8	8.7	...	...	...	...	...	...	...	SA-442
18.8	...	...	...	...	...	...	...	...	...	...	...	SA-455
18.3	...	...	...	...	...	...	...	...	...	...	...	SA-455
17.5	...	...	...	...	...	...	...	...	...	...	...	SA-455
13.8	13.3	12.1	10.2	8.4	6.5	4.5	2.5	...	...	...	...	SA-515
15.0	14.4	13.0	10.8	8.7	6.5	4.5	2.5	...	...	...	...	SA-515
16.3	15.5	13.9	11.4	9.0	6.5	4.5	2.5	...	...	...	...	SA-515
17.5	16.6	14.8	12.0	9.3	6.5	4.5	2.5	...	...	...	...	SA-515
13.8	13.3	12.1	10.2	8.4	6.5	4.5	2.5	...	...	...	...	SA-516
15.0	14.4	13.0	10.8	8.7	6.5	4.5	2.5	...	...	...	...	SA-516
16.3	15.5	13.9	11.4	9.0	6.5	4.5	2.5	...	...	...	...	SA-516
17.5	16.6	14.8	12.0	9.3	6.5	4.5	2.5	...	...	...	...	SA-516

FIG. A-8 (2) Maximum Allowable Stress Values for Carbon and Low Alloy Steel [22]

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